Volume and taper equations for Sitka spruce (Picea sitchensis (Bong.) Carr.), Norway spruce (Picea abies (L.) Karst.) and White spruce (Picea glauca (Moench) Voss) in Iceland.

Volym och avsmalningsfunktioner för sitkagran (Picea sitchensis (Bong.) Carr.), gran (Picea abies (L.) Karst.), och hvitgran (Picea glauca (Moench) Voss) på Island.

Lárus Heiðarsson

Arbetsrapport 424 2014
Examensarbete 30hp A1E

Handledare:
Kenneth Nyström

Sveriges lantbruksuniversitet
Institutionen för skoglig resurshushållning
901 83 UMEÅ
www.slu.se/srh
Tfn: 090-786 81 00

ISSN 1401–1204
ISRN SLU–SRG–AR–424–SE

Volume and taper equations for Sitka spruce (Picea sitchensis (Bong.) Carr.), Norway spruce (Picea abies (L.) Karst.) and White spruce (Picea glauca (Moench) Voss) in Iceland.

Volym och avsmalningsfunktioner för sitkagran (Picea sitchensis (Bong.) Carr.), gran (Picea abies (L.) Karst.), och hvitgran (Picea glauca (Moench) Voss) på Island.

Lárus Heiðarsson

Master thesis in Forest Management at the dept of Forest Resource Management

EX 0492, A1E
Supervisor: Kenneth Nyström, SLU, Dept. of forest resource management
Examiner: Göran Ståhl, SLU, Dept. of forest resource management

Sveriges lantbruksuniversitet
Institutionen för skoglig resurshushållning
Utgivningsort: Umeå
Utgivningsår: 2014

ISSN 1401–1204
ISRN SLU–SRG–AR–424–SE

Summary

The aim of this study was to evaluate different types of volume and taper equations that can be used to predict single-tree stem volume and stem diameter at any given height along the tree stem for plantation grown Sitka spruce (*Picea sitchensis* (Bong.) Carr.), Norway spruce (*Picea abies* (L.) Karst) and White spruce (*Picea glauca* (Mounch) Voss) in Iceland. A number of published tree volume equations were tested and modified to predict the total stem volumes over bark but three logarithmic equations were taken for more in-depth analysis. Three taper equations were tested. Two variable-exponent equations (Kozak 1997, Kozak 2004) and one exponential equation described by Biging (1984). Data from a total of 617 sample trees were used in this study, collected from stands in various parts of the country and present different types of stands growing in different soil types and cover most of the site conditions suitable for forestry in Iceland. To fit the regression model for the volume equations an ordinary least-squares (OLS) method was used. Because the construction of taper equations requires longitudinal data or multiple measurements on individual trees, and violates the assumption of independence between observations, a mixed effects approach was used to model the within tree autocorrelation. Volume equation [5] which has breast height diameter (D), tree height (H) and (H-1.3) as independent variables gave the best results based on fit and validation statistics and are most suitable for all three species. In diameter prediction a modified version of the Biging (1984) equation gave the best results based on fit and validation statistics and is most suitable for all three species. In volume prediction the Biging (1984) equation showed some bias in predicting volume of small trees and the same was noticed for the equation developed by Kozak (1997). The equation developed by Kozak (2004) seems to be more flexible in predicting the volume of small trees as well as bigger trees and should give the best results in volume prediction among the taper equations.

Sammanfattning

Syftet med denna studie har varit att utvärdera olika typer av volym och avsmalnings funktioner som kan användas för att förutsäga stamvolym och stamdiameter vid en viss höjd längs trädet för planterad Sitka gran (Picea sitchensis (Bong.) Carr.), gran (Picea abies (L.) Karst) och Vit gran (Picea glauca (Mounch) Voss) på Island. Ett antal publicerade volym samband testades och modifierades för att prediktera den totala stamvolymen på bark. De tre avsmalningsmodeller som utvärderades i studien var två variablaexponent modeller (Kozak 1997, Kozak 2004) och en exponentiell modell beskriven av Biging (1984). Data från totalt 617 provträd ligger till grund för denna studie. Materialet har samlats in från bestånd i olika delar av landet och representerar olika ståndorter och omfattar merparten av markförhållanden lämplig för skogsbruk på Island. Skattningarna av parametrarna i volymsfunktionerna har utförts med regressionsanalys enligt minsta-kvadrat metoden(OLS). Eftersom avsmalningsfunktionerna baseras på data med flera mätningar på ett enskilt träd längs stammen kan vi inte utan vidare göra antagandet om oberoende mellan observationer i datat. För avsmalningsfunktionerna användes därför en blandad regressionsmodell med fixa parametrar och där trädindividen specificerades som en slumpmässig effekt. Volym ekvation [5], som har brösthöjdsdiameter (D), trädhöjd (H) och (H – 1.3) som oberoende variabler gav det bästa resultatet baserat på valideringsstatistiken och residualstudier för alla tre arterna. För avsmalningsfunktionerna gav Kozak 2004 det bästa resultatet för alla tre arterna. Skillnaderna mellan modellerna var dock små.

Contents

Introduction

Iceland has a short forest history and legislation was first approved in 1907 to protect the remaining woodlands and to create new forests (Aradottir & Eysteinsson 2005). During the past 60 years, emphasis has been on afforestation through planting trees (Eysteinsson 2009). The main task has been to find the right species and provenances that are adapted to Icelandic climate and growing conditions. As the forests have become older the need to introduce and evaluate equations for estimating tree growth, taper and stem volume for different tree species has become more evident. Studies on stem form have long been basic tasks in forest mensuration rescearch (c.f. Gray 1956, Larson 1963, Assmann 1970, Karlsson 2005). For forest mensuration, stem form is of interest in determining the volume and value of the whole stem or a part of it (Lappi 1986). As wood consumption from Icelandic forests continues to increase as they grow older and the growing stock becomes of commercial interest, determination of the extent, structure and increment of forest timber resources, as well as following changes in them, will be a major task in the future for Icelandic foresters. Stem content can be expressed in volume or weight terms or as an estimate of end product output from some manufacturing process (Clutter *et al.* 1983). Volume equations are used to estimate tree and stand volume, and have played a crucial role in forest inventories and management for more than a hundred years (c.f. Jonson 1928, Näslund 1940, Laasasenaho 1982, Brandel 1990). Taper equations were introduced much later, their advantage over volume equations being that they can describe changes in diameter along stem height, and therefore provide estimates of dimensions of logs that might be cut from stems. Volume equations to estimate the total stem volume of the most common species used in Icelandic forestry have been published by Norrby (1990), Snorrason & Einarsson (2006), Bjarnadottir *et al.* (2007) and Juntunen (2010), but equations to predict tree taper have only been published for lodgepole pine (*Pinus contorta* Dougl.) and Siberian larch (*Larix sibirica* Lebed.) by Heidarsson & Pukkala (2011). The tree species studied in the present study play different roles in Icelandic forestry. Sitka spruce is today one of the main tree species, growing well in various climate and soil conditions covering an area around 5000 hectares (Snorrason 2014). Norway spruce was among the most planted tree species in Iceland from 1950 to 1975 covering an area around 750 hectares (Snorrason 2014). White spruce is a less important species but has been widely planted for some years. Both Norway spruce and white spruce are more sensitive to the variable winter climate in Iceland than Sitka spruce. For average growth, they cannot be planted near the coast and need both shelter and fertile soil.

Volume equations

Volume equations are used to estimate tree and stand volume and have played an important role in forest research. A multitude of equations has been published in forest literature (c.f. Schumacher & Hall 1933, Spurr 1952, Clutter *et al.* 1983, Avery & Burkhart 2002, Hjelm 2011). Because of inherent morphological differences among tree species, it is generally necessary to develop separate standard volume equations for each species or closely related species group (Burkhart & Gregoire 1994). Usually diameter at breast height (D at 1.3 meters height) and total tree height (H) are the most important independent variables that are commonly used to determine the value of the individual tree volume (Hush 1982) and tend to account for the greatest proportion of the variability in the volume of a tree. According to Laar and Akça (2007) volume equations can be classified based on the number of predictor variables of the volume equation:

• Single-entry volume equation (one-way table)
• Multiple-entry volume equation (two-way table and three-way table)

Single entry volume equations were first developed towards the end of 19th century for all aged forests in France and adapted for management of mixed uneven aged forests in Switzerland (Laar and Akça 2007). Normally, diameter at breast height (D) or basal area (G) is required for constructing a single entry volume equation. Such equations are generally restricted to a local area because trees in a given diameter class can vary in there heights and forms, especially those from different sites. According to Philip (1994) volume equations of this type have to be restricted to a small range of diameters in a specific stand at a specific age. Multiple entry volume equations include both double entry and triple entry volume equations. Double entry volume equations often referred to as „standard volume equations“ use both D and H as independent variables. Triple entry volume equations have a third variable, such as diameter at a specified upper height, crown height above ground or some other indicator of the form or shape of the tree. Some papers reported that the addition of a third predictor variable reduced the amount of unexplained variation and improved the accuracy of volume estimates, while other studies found that the addition of a third predictor variable did not significantly improve the quality of prediction (Laar and Akça 2007). Double entry volume equations are probably the most common form of volume equations (Philip 1994). Equations to estimate the total stem volume of the most common species used in Icelandic forestry have been published by Norrby 1990, Snorrason & Einarsson 2006, Bjarnadottir et al. 2007 and Juntunen 2010. All the equations are multiplicative type, double entry equations based on the natural logarithm of the dependent variable, using D and H as independent variables and volume over bark as the dependent variable.

Norrby (1990) presented local equations for Siberian larch in Hallormsstadur, located in eastern Iceland. The relationships were based on 100 felled sample trees and the equations should not be used outside the Hallormsstadur area or outside a diameter interval of 4 – 20 cm.

Later, Snorrason and Einarsson (2006) developed volume equations for eleven tree species in Iceland. These species are downy birch (*Betula pubescens*), rowan (*Sorbus aucuparia*), feltleaf willow (*Salix alaxensis*), dark-leafed willow (*Salix myrsinifolia*), black cottonwood (*Populus trichocarpa*), Sitka spruce (*Picea sitchensis*), Engelmann spruce (*Picea engelmannii*), white spruce (*Picea glauca*), Norway spruce (*Picea abies*), lodgepole pine (*Pinus contorta*), and Siberian larch (*Larix sibirica*). The equations for Norway spruce were constructed using data from 16 sample trees and a joint equation was developed for

Sitka spruce and white spruce using data from 56 observations. From that research 13 Norway spruce sample tree and all the Sitka and White spruce data are included in the development of the new equations here and separate equations were made for Sitka and White spruce.

Volume and biomass equations for young larch plantations were developed by Bjarnadottir et al. (2007). The data were from a twelve year old plantation and the independent variables for volume calculation are total tree height and diameter at 0.5 meters height.

Juntunen (2010) developed volume equations for lodgepole pine using data from 87 trees collected from sample plots all around the country.

Sweden has a long history of research regarding form and volume of tree stems. In 1932 Petterson introduced correlation analysis in forest research in Sweden. This had a tremendous effect on the possibilities of constructing volume equations by using the least square method (Brandel 1990). Näslund (1940, 1947) presented two kinds (simple and advanced) of additive stem volume equations for Scots pine, Norway spruce and Birch in Sweden. The equations used the form factor as a dependent variable. The equations were transformed into volume equations by multiplication of the tree height and basal area at breast height (Brandel 1990). The simpler equation use D and H as independent variables and the advanced equations use also crown height and bark thickness as additional variables. The equations have been frequently used in Sweden (Hjelm 2011).

Eriksson (1973) presented volume equations for ash (*Fraxinus excelsior* L.), European aspen (*Populus tremula* L.), common alder (*Alnus glutinosa* (L). Gaertn.) and lodgepole pine (*Pinus contorta*, Douglas). The equations are of a multiplicative type using D and H as independent variables. In some of the equations, Eriksson used crown height above ground and crown length defined as percent of tree height.

Brandel (1990) published multiplicative volume equations for Scots pine, Norway spruce and Birch in Sweden. The equations used volume as the dependent variable and D and H as independent variables. Upper height diameter at 6 meter height, crown height above ground and bark thickness at breast height are additional variables that can be added to the base equation.

Different types of volume equations for common alder and grey alder (*Alnus incana* (L.) Moench) have recently been published by Johansson (2005). The equations are of multiplicative, additive and logarithmic type using D and H as independent variables and crown height included in some of the equations tested.

Volume equations for poplar (*Populus sp.*) growing on farmland in Sweden have recently been published by Hjelm (2011). A number of published equations were tested and two equations were constructed. One of the constructed equations is a double entry equation and the other multiple entries with upper height diameter as an additional variable. Including the third variable in the equation, increased the performance of the prediction notably (Hjelm 2011).

Taper equations

Taper can be defined as the rate of narrowing in diameter along the tree stem of a given form (Gray 1956). Taper equations are often the basis of computer algorithms for calculating stem diameter at any height along the stem to predict merchantable volume to any limit, a prerequisite for successful forest planning and management (Kublin et al. 2008). To date, no single theory or model exists that adequately explains the variation in stem form for all species (Newnham 1988). According to Kozak (2004), taper equations provide forest managers estimates of (1) diameter at any point along the stem, (2) total stem volume, (3) merchantable volume and merchantable height to any top diameter and from any stump height, and (4) individual volumes for stem sections of any length and at any height from the ground. The stem form of a tree is strongly influenced by its crown size and position. Changes in the size of the living crown, the distribution of branches within the crown and the length of the branch-free bole are attributes that create variations in stem taper (Larson 1963). So when two trees with the same total height, but different crown length are compared, the tree with the longer crown will exhibit larger breast height diameter and greater rate of taper on the lower stem than the tree with shorter crown length (Gray 1956, Muhairwe 1994). However incorporating crown dimensions into taper equations in previous studies have shown mixed fit results. In most of the basic forest mensuration text it is generally assumed that a tree stem can be divided in three geometric shapes. Closest to the ground the bole portion is assumed to be a neiloid, the middle portion a paraboloid and the upper portion a cone (Hush et al. 1972). Because of that, and stem taper curves are predicted to various sizes of trees, a flexibility of the stem taper prediction equation is an essential factor and must be taken into account in model construction (Eerikainen 2001). Taper equations are generally based on a trees diameter at breast height (D), total height (H) and the height above ground (*h*) (the point where the diameter will be predicted) as independent variables. The importance of taper equations is demonstrated by the high number of equations published and used, varying in complexity and various tree species. Many studies in this field have involved polynomials of order two or greater (e.g. Bruce et al. 1968, Kozak et al. 1969, Goulding & Murray 1976, Laasasenaho 1982) were the stem profile is describe by a single equation. According to Sterba (1980) the weakness of this equation type has been the inability to characterize the lower portion of a tree with significant basal swelling. Another type is the segmented polynomial equation, which uses different equations for various parts of the stem and then mathematically joining them to produce an overall segmented equation (e.g. Max & Burkhart 1976, Demaerschalk & Kozak 1977). Later, variable-exponent taper equations where introduced, which use changing exponents to describe the different shapes of a bole from the ground to the top (e.g. Kozak 1988, 2004, Newnham 1988, 1992). These equations enable the exponent to change with relative tree height expressed as (h/H), which allows a single equation to describe the stem profile. Other types of taper equations can also be found in the literature, such as trigonometric (Thomas & Parresol 1991), exponential (Biging 1984) and nonparametric (Lappi 2006) equations. Variable exponent taper equations have been found to be superior to segmented and simple models for estimating stem diameters and volumes (Kozak 1988, Newnham 1992, Muhairwe 1999, Rojo et al. 2005). However, they cannot be integrated analytically to calculate total stem or log volumes (Diéguez-Arunda et al. 2006), which must be estimated instead from calculated diameters and lengths by numerical integration (Kozak 1988). The first equations for predicting tree taper in Iceland and Sweden have recently been published (Heidarsson & Pukkala 2011, Hjelm 2011). The Icelandic equations were made for Siberian larch and lodgepole pine and are variable-exponent taper equations. The Swedish equations were made for poplar and simple, segmented and variable-exponent equations were tested. The variable-exponent equation had the best fit statistics according to bias and RMSE.

Objectives of the study

The objective of this study was to develop and evaluate volume and taper equations that can be used to predict single-tree volume and stem diameter at any given height along the tree stem for Sitka spruce, Norway spruce and White spruce, commonly used tree species in Icelandic forestry. The objective was also, to compare existing volume equations with the best volume equations from this study and evaluate the performance of the existing equations with the data set from this study.

Material and Methods

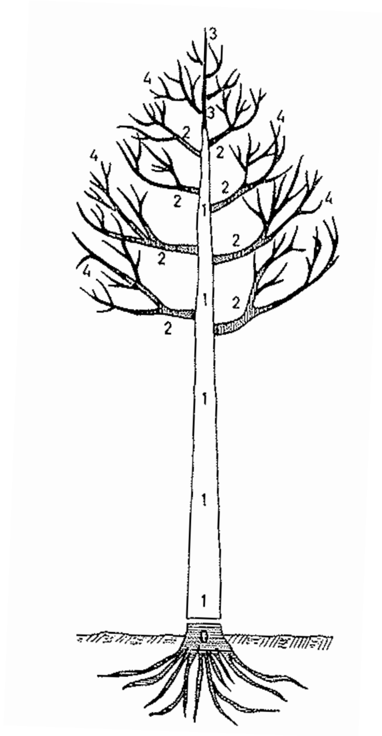
Data collection

Data from 617 sample trees were used in this study. The material used was selected from different research projects were tree volume measurements of Sitka spruce, Norway spruce and White spruce were sampled between 2001 and 2009. The data consist of total 6106 individual tree diameters (over bark) measured at different intervals up to the tip, diameter at breast height over bark (1.3 m above ground; D), total height (H) above stump, and stem volume (V) above stump (Table 1). Most of the data were from Iceland Forest Service research sites located in different parts of the country, planted between 1942 and 1983 and represent different climate regions around the country, different types of stands growing on different soil types and cover most of the site conditions suitably for forestry on Iceland.

The data used in present study originated from the following data sources;

- Gpot, data from a project dealing with the growth potential of eleven tree species in Iceland described in Snorrason & Einarsson (2006). These plots were evenly spread around the whole country.
- PtH, data from a provenance trial established in 1958 at Hallormsstaður in the eastern part of the country.
- PtS, data from a provenance trial established in 1958 at Stálpastaðir in the western part of the country.
- TP, data originating from thinning trials of Sitka spruce located in Haukadalur and Tumastaðir in southern Iceland.
- PSP, data from permanent sample plots collected during 2002- 2010.

On sample trees from the Gpot data, tree diameters over bark were measured at different relative heights; every 5% of height under breast height (1.3 m) and every 10% of total height over breast height. For TP and PSP, tree diameters over bark were measured at the following relative heights, which are given as percentages of the total tree height: 1, 2.5, 5, 7.5, 10, 15, 20, 30, 40, 50, 60, 70, 80, 90 and 95%. For PtH- and PtS data, diameters over bark were measured at the height of 0.5 m and then at one meter intervals up to the tip. In figure 1, the volume of different tree parts is demonstrated and may be given with or without bark. In this study the total stem volume is of interest (number four).



1.	0 + 1 + 2 + 3 + 4	Total tree volume
2.	1 + 2 + 3 + 4	Total stem plus total branches
3.	1 + 2	Merchantable tree volume
4.	1 + 3	Total stem volume
5.	2 + 4	Total volume of branches
6.	3 + 4	Unmerchantable tree volume

Figure 1. Total tree volume and volume of different tree parts (from Loetsch & Haller 1973).

Volume determination on felled sample trees was based on measurements of diameter and length. The stem was subdivided into sections and cross-sectional area was measured in the middel of each section. For each section, volume was calculated using Huber's formula, were the section is asumed to be cylinder [1]. The total stem volume was derived by adding all the sections together. Summary statistics and relevant tree characteristics are provided in Table 1 and 2.

$$V = g_m L \qquad [1]$$

V = volume of the log

g_m = cross-sectional area at log midpoint

L = log length

Table 1. Total number of sample trees from different research projects. Gpot data are from the 2001 country-wide research; PtH data are from the provenance trial in Hallormsstaður; PtS data are from the provenance trial in Stálpastaðir; TP data are from thinning plots and PSP data are from permanent sample plots.

	Gpot	PtH	PtS	TP	PSP	Total
Norway spruce	13	151	132		22	318
Sitka spruce	24	50	52	22	45	193
White spruce	9	41	41		15	106
Total	**46**	**242**	**225**	**22**	**82**	**617**

Table 2. Summary statistics for tree attributes for the species in the present study.

Norway spruce (n = 318)				
Variable	Mean	S.D.	Maximum	Minimum
D (cm)	9.9	4.2	22.7	1.0
H (m)	7.3	2.2	13.1	1.7
Age (years)	47.9	2.9	53.0	27.0
Volume (dm^3)	41.6	36.0	184.0	0.5
Sitka spruce (n = 193)				
Variable	Mean	S.D.	Maximum	Minimum
D (cm)	14.5	6.4	28.1	2.8
H (m)	9.9	3.1	16.3	2.6
Age (years)	46.5	6.9	62.0	21.0
Volume (dm^3)	113.8	99.7	418.8	2.0
White spruce (n = 106)				
Variable	Mean	S.D.	Maximum	Minimum
D (cm)	11.01	5.1	26.0	2.5
H (m)	7.5	2.2	12.5	2.6
Age (years)	46.9	4.2	49.0	31.0
Volume (dm^3)	51.5	53.0	280.0	1.9

D is diameter at 1.3 m above ground level; H is total height; S.D. is standard deviation and age is planting age.

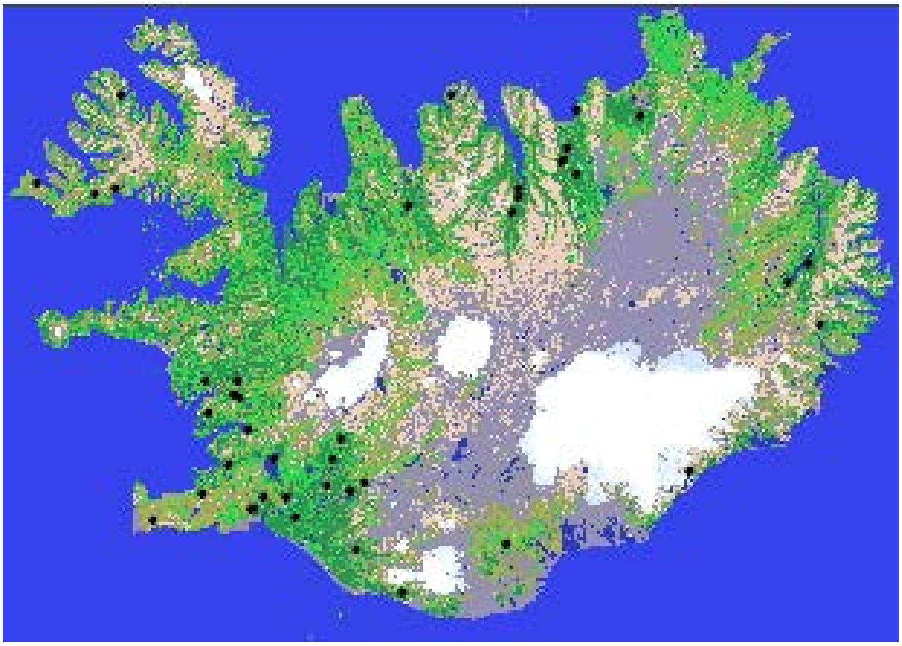


Figure 2. Geographical location of the study sites.

Methodological considerations and evaluation of the dataset

A part of the dataset had a spatially hierarchical structure, i.e., several trees were measured from the same plot and trees from the same sample plot tend to resemble each other more than average. This hierarchical structure might result in dependence between observations within a certain plot or site. In order to check if there was dependence in the data-set a mixed model approach was applied where the stands and plots within stands were included as random effects (parameters) in the model. The mixed model approach didn´t reveal any significant effects of sites or plots in the model. The different sites represented in the study were also included as indicator variables in the OLS case. However, the site differences were not significant in the data set.

For the volume equations the assumption of constant variance of the dependent variable in the regression analyses did not hold because the variation in volume of trees was dependent of the tree size (c.f. Korhonen & Eerikäinen 2001). One of the key assumptions of regression (with OLS) is constant variance; standard estimation methods being inefficient when the errors have non-constant variance (are heteroscedastic). Even though the estimates of the parameters may be unbiased, their variance (and standard errors) may be biased or inconsistent. To overcome that, logarithmic transformation was used to transform the dependent variable (V) to obtain constant variance of the errors (residuals) and makes it possible to estimate the parameters by linear regression. This is a common procedure when describing volume or biomass in equations with diameter and/or height as the independent variables (Pardé 1980). Transformation the logarithmic predictor back into the arithmetic scale results in a biased predictor (Lappi 1991). The correction factor for the logarithmic equation is approximately half of the estimated error variance of the equation (Baskerville 1972). The correction term is an easy statistical tool to extract a systematic bias and it should always be used with logarithmic transformations of allometric equations

(Sprugel 1983). In the present study the results were corrected for logarithmic bias according to Finney (1941), equation [2].

$$Q = S^2/2 \qquad [2]$$

Where S is the residual standard deviation about the regression and Q is the correction factor to correct for logarithmic bias. Since there were no evident correlations in the volume data after transformation of the dependent variable, it was appropriate to fit the regression model using the ordinary least-squares (OLS) method.

Construction of taper models requires information of longitudinal data or multiple measurements on individual trees (Lindström & Bates 1990). Where several measurements of diameter at several heights from the ground are used from a single tree the measurements or error terms are said to be serially correlated, or autocorrelated (Kozak 1997). This autocorrelation violates the assumption of independence between observations that is one important key to obtaining an unbiased estimate of the covariance matrix in regression (Valentine & Gregoire 2001). Multicollinearity is common in empirical relationships i.e. taper equations, and refers to the existence of a high degree of correlation among several independent variables. This occurs when too many variables have been included in a model and a number of different variables measure similar phenomena (Rojo et al. 2005). The existence of multicollinearity is not a violation of the assumptions underlying the use of regression, and therefore does not seriously affect the predictive ability of the model (Kozak 1997). However, the presence of multicollinearity may inhibit the usefulness of the results. According to Kozak (1997), the following problems occur when multicollinearity exists: (1) small changes in the data can produce significant changes in the parameter estimates, (2) regression coefficients have high standard errors, which affect the significance level of the corresponding independent variable, and (3) the regression coefficients may have a wrong sign and/or an unreasonable magnitude. One of the main sources of multicollinearity is the use of overcomplicated models that include cross product terms, something that is common in many taper equations. To address the problem with autocorrelation and multicollinearity a mixed effects modelling technique was used in the present study. This modelling approach has gained broad acceptance in forest growth and yield modeling to achieve better local prediction and to handle residual autocorrelation from repeatedly measured data (Trincado & Burkhart 2006), and estimates the covariance matrix of correlated data by allowing a non-constant correlation among the observations (Lindström & Bates 1990). Mixed effects models contain both fixed effects parameters common to all subjects as in traditional regression and random effects parameters specific for each subject accounting for various sources of heterogeneity and randomness in the data caused by known and unknown factors (Vonesh & Chinchilli 1997).

Volume equations

Tree stem volume (V) is usually considered to be a function of the tree's diameter at breast height (D), height (H), and form factor (F). A common expression for tree volume equations is:

$$V = f(D, H, F) \qquad [3]$$

The form factor is rarely used in tree volume model construction. Even though form (F) is required in some formulae for the volume of a tree, it is not a truly independent variable. Like volume, the form factor is usually estimated from other measurements of a tree's dimensions and can be neither measured directly nor calculated without first measuring the volume (Philip 1994). A number of published tree volume equations were compared in the present study and modified to predict the total over bark stem volumes and the following second entry logarithmic equations were taken for more in-depth analysis. These were:

$$lnV_i = \beta_0 + \beta_1 lnD_i + \beta_2\, lnH_i + \varepsilon_i \qquad [4]$$

$$lnV_i = \beta_0 + \beta_1 lnD_i + \beta_2\, lnH_i + \beta_3 ln(H_i - 1{,}3) + \varepsilon_i \qquad [5]$$

$$lnV_i = \beta_0 + \beta_1 lnD_i + \beta_2\, lnH_i + \beta_3 ln(H_i - 1{,}3) + \beta_4 ln(D_i + 20) + \varepsilon_i \qquad [6]$$

Where ln V_i is the natural logarithm (ln) of stem volume, β_0 to β_4 are parameters to be estimated, ln(D_i) is log diameter at breast height (D), ln(H_i) is log total height, ln(H_i-1.3) is log total height minus 1.3, ln(D_i+20) is log diameter at breast height (D) plus 20 and ε_i is the random error term of the equation, which is assumed to be independent and identically distributed with mean equal to zero and constant variances.

Taper equations

A tree's taper is the rate of change of stem diameter with increased height along the tree, and can be expressed as an equation of diameter at breast height (D), total height (H) and upper stem height (h). The use of total tree height in taper equations is very important because changes in tree shape and tree taper are characterized by changes in total tree height and diameter (Muhairwe 1994). It also enables conditioning of the model such that when the height above the ground (h) of prediction is equal to the total height, the predicted diameter along the stem is zero, i.e., D=0 when h=H (Muhairwe 1999). The common expression for functional form of taper equations is:

$$d = f(D\ H\ h) \qquad [7]$$

Three taper equations were compared for all three tree species. Two variable-exponent taper equations [8] (Kozak 1997) and [9] (Kozak 2004) and one exponential equation [10] described by Biging (1984). Equation [8] was found to be the best for Calabrian pine (*Pinus brutia*) in Syria among the 32 equations tested by de Miguel et al. (2011). Equation

[9], also developed by Kozak, is newer and should have a lower multicollinearity than equation [8] and the Biging equation has the advantage of being simple, having only three parameters. The original equations are presented in table 2. According to Kozak (2004) equation [9] can be modified or improved for local conditions and different species by changing the terms in the exponent, base of the exponent or the multiplier of the base (e.g., $b_1 D^{b_2} H^{b_3}$).

Table 2. Original taper equations tested in this study.

Equation [8].

$$d_{ij} = b_1 D_j^{b_2} H_j^{b_3} X^{b_4} X^{0.1} + b_5 q_{ij}^4 + b_6 arcsin\left(1 - \left(q_{ij}^{\frac{1}{2}}\right)\right) + b_7 \left(\frac{1}{exp^{\frac{D_j}{H_j}}}\right) + b_8 D_j^X \times \left(1 + \varepsilon_{tij}\right) + \varepsilon_{rij}$$

Equation [9].

$$d_{ij} = b_1 D_j^{b_2} H_j^{b_3} X^{b_4} q_{ij}^4 + b_5 \left(\frac{1}{exp^{\frac{D_j}{H_j}}}\right) + b_6 X^{0.1} + b_7 \left(\frac{1}{D_j}\right) + b_8 H_j^Q + b_9 X \times \left(1 + \varepsilon_{tij}\right) + \varepsilon_{rij}$$

Equation [10].

$$d_{ij} = D_j \times (b_1 + b_2) \times ln\left(1 - \lambda \times (q_{ij})^{b3}\right) \times \left(1 + \varepsilon_{tij}\right) + \varepsilon_{rij}$$

Where

$$\lambda = 1 - exp\left(1 - \frac{b_1}{b_2}\right)$$

For all the equations, d_{ij} is the predicted diameter (d, cm) at relative height i (m) for tree j. D_j is diameter at 1.3 m for tree j, H_j is total tree height (m) for tree j, q_{ij} is relative height i (h_i/H) for tree j. In equation [8], X is (1-$q_{ij}^{1/2}$)/(1-(1.3/H)$^{1/2}$), and Q is (1-$q_{ij}^{1/2}$). In equation [9], X is (1-$q_{ij}^{1/3}$)/(1-(1.3/H)$^{1/3}$), and Q is (1-$q_{ij}^{1/3}$). In this study a modified version of the Biging equation was tested, where b_1 is estimated with A0+A1×ln(H_j/D_j), b_2 is D0+D1×ln(H_j/D_j) and b3 is C0+C1×(1.3-H_j). For all the equations the parameters b_1 to b_9, A0, A1, D0, D1, C0 and C1 are regarded as fixed. ε is the random error component in the model. The random variation in d_{ij} was separated into: (i) a multiplicative component of random variation within the tree (ε_{tij}), and (ii) a additive component to the unexplained residual variation (ε_{rij}). The distributional assumptions of the error terms were: $\varepsilon_{tij} \approx NID(0, \sigma^2_{\varepsilon_{tij}})$, $\varepsilon_{rij} \approx NID(0, \sigma^2_{\varepsilon_{rij}})$, and tied as: $\sigma^2_{\varepsilon ij}$= f(x)×$\sigma^2_{\varepsilon tij}$+$\sigma^2_{\varepsilon rij}$.

Statistical procedures

All regression analyses for both volume and taper equations were carried out with the SAS statistical software package version 9.2. The primary analysis of the volume equation used the MIXED procedure, while further analysis of the volume equations used the ordinary OLS REG procedure for parameter estimation and model fit. The NLMIXED procedure was used in parameter estimation and fit of the taper equations.

Evaluating equations

All the equations were evaluated and compared on the basis of the degree of explained variance (R^2), bias (B) for systematic errors and root mean squared error (RMSE) as indication of precision. Distributions of residuals were also examined visually for identify any systematic trends. These statistics were defined as:

$$R^2 = 1 - \sum(y_i - \hat{y}_i)^2 / \sum(y_i - \bar{y})^2$$

$$\text{Bias} = \frac{\sum(y_i - \hat{y}_i)}{n}$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=0}^{n}(y_i - \hat{y}_i)^2}{n-1}}$$

Where $y_i, \hat{y}_i$ and $\bar{y}$ are measured, predicted and average values of the dependent variable and n is the total number of observations used to fit the model. Although single indices of overall prediction (R^2, Bias and RMSE for $\hat{d}$ and $\hat{v}$) are good indicators of the effectiveness of the equations, they may not show the best equation for practical purposes (Muhairwe, 1999). Therefor the relative differences (RD $= \frac{(v_i - \hat{v}_i)}{\hat{v}_i}$) of the estimated and predicted volume equations were evaluated for any systematic trend in different diameter classes and at different relative heights along the stem. The taper equations were also compared using the Akaike‘s information criterion (AIC) and the Schwarz‘s Bayesian information criterion (BIC) derived from the fit statistics, two widely accepted methods when comparing models with datasets affected by autocorrelation and different number of parameter (Burnham & Anderson 1998). If any of the parameters in the equations was non-significant according to the t-test ($p > 0.05$) the corresponding predictor was dropped from the equation and a simpler version of the equation was fitted to the data.

A comparison was made of “true“ total stem volume measured from sample trees and the different taper equations present in the study. In order to obtain an estimated total volume for each individual tree, the volume was derived by numerical integration of the taper equation.

Results

Volume equations

Norway spruce

The estimated parameters of fitted volume equations tested for Norway spruce are presented in table 3. Equations [5] and [6] had the same R^2, little higher than equation [4]. For equation [6] the parameter β_4 was not significant, indicating any better fit statistics with an additional diameter variable into equation [5].

Table 3. Parameter estimates and fit statistics for Norway spruce. All parameters are significant at 5 % level ($p < 0.05$) except β_4.

Equation	β_0	β_1	β_2	β_3	β_4	R^2
4	-2.0281	1.6491	0.8629			0.9852
5	-3.5322	1.6884	2.9168	-1.5059		0.9923
6	-3.5360	1.6879	2.9160	-1.5052	0.0016	0.9923

Table 4. Mean value of bias (dm^3) and RMSE (dm^3) in different diameter (D) classes and over all fit for equations [4], [5], [6] and existing equation for Norway spruce. The lowest values of bias and RMSE in different diameter classes are marked bold.

Diameter		Equation 4		Equation 5		Equation 6		Existing equation	
Class	n	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
< 5	36	-0.2094	0.5442	**-0.0407**	**0.3786**	-0.0411	0.3786	-2.563	0.5738
5-7.5	62	-0.705	1.038	**0.247**	**0.986**	0.246	0.986	-1.313	1.064
7.5-10	65	-0.684	2.525	**0.190**	2.388	0.186	2.388	-2.200	**2.337**
10-12.5	79	1.670	4.034	**1.109**	**3.629**	1.101	3.629	-1.223	3.914
12.5-15	38	4.59	6.52	0.935	6.64	0.921	6.64	**-0.300**	**6.39**
15-17.5	24	8.08	10.84	**0.391**	**8.18**	0.369	8.18	1.34	10.23
17.5-20	7	5.03	10.31	**-2.05**	**8.97**	-2.08	8.97	-4.78	9.41
> 20	7	14.40	8.77	2.56	12.73	2.51	12.73	**1.59**	**9.89**
All	318	1.734	6.05	**0.546**	**4.47**	0.544	4.47	-1.066	4.75

Results for validation of bias and RMSE for the equations in different diameter (D) classes and over all fit are presented in table 4. Equations [5] and [6] perform better than equations [4], in most of the diameter (D) classes. The estimated overall value of bias and RMSE for the existing equation (Snorrason & Einarsson 2006) tested on data from the present study are a little higher than equations [5] and [6] but lower than equation [4] (table 3). When the existing equation is compared with equation [5] in different diameter classes the values are marginally higher in most of the diameter classes for the existing equation. When the relative difference (RD) for equation [5] is compared to an existing equation made by Snorrason & Einarsson (2006) it is clear that the equation [5] has a better fit to the data (Figure 3). The existing equation has a poorer fit in diameter classes smaller than 10 cm but the bias and variation is very similar in other classes.

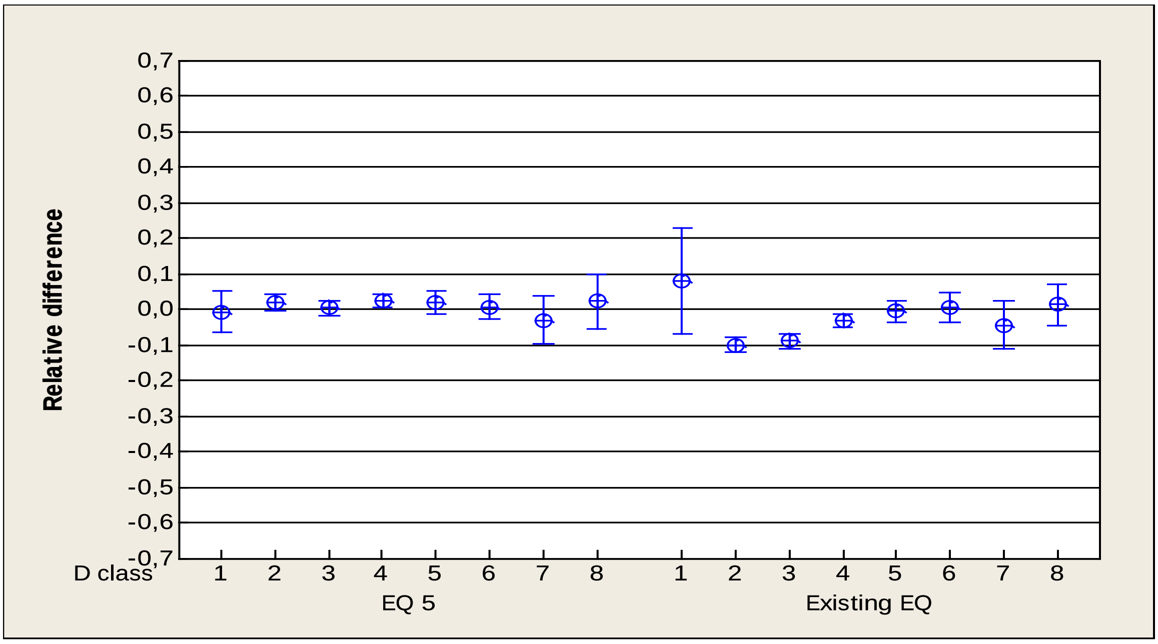


Figure 3. Residuals expressed as relative differences ($(v - \hat{v})/\hat{v}$) between observed and predicted volume for Norway spruce presented in different diameter (D) classes with 95% confidence intervals. The classes are 1: D <5 cm, 2: <7.5 cm, 3: <10.0 cm, 4: <12.5 cm, 5: <15.0 cm, 6: <17.5 cm 7: <20.0 cm and 8: >20.0 cm. Comparison between equation [5] and the existing equation by Snorrason & Einarsson (2006).

Sitka spruce

The estimated parameters of fitted volume equations tested for Sitka spruce are presented in table 5. Equations [5] and [6] have the same R^2, marginally higher than equation [4]. Parameter β_4 was not significant for equation [6] indicating any better fit statistics with an additional diameter variable into equation [5].

Table 5. Parameter estimates and fit statistics. All parameters are significant at 5 % level ($p < 0.05$) except β_4.

Equation	β_0	β_1	β_2	β_3	β_4	R^2
4	-2.4533	1.7141	1.0056			0.9884
5	-3.2870	1.7119	2.5908	-1.3059		0.9892
6	-3.8570	1.6064	2.3418	-1.0899	0.2694	0.9892

Results for validation of bias and RMSE for the equations in different diameter (D) classes and over all fit are presented in table 6. The lowest values in every diameter (D) class are marked bold. Equations [4], [5] and [6] perform similarly in most of the diameter (D) classes but equation [5] has the lowest value more often than the other equations. The value of bias and RMSE for the existing equation (Snorrason & Einarsson 2006) tested on this study data are little higher than equations [5] and [6] but little lower then equation [4] regarding overall bias and RMSE (table 6).

Table 6. Mean value of bias (dm^3) and RMSE (dm^3) in different diameter (D) classes and over all fit for equations [4], [5], [6] and existing equation for Sitka spruce. The lowest values of bias and RMSE in different diameter classes are marked bold.

Diameter		Equation 4		Equation 5		Equation 6		Existing equation	
Class	n	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
< 5	10	0.116	0.833	**-0.0562**	**0.731**	-0.103	0.755	-4.50	0.794
5-7.5	20	**0.650**	**3.586**	0.810	3.612	0.763	3.598	1.270	3.615
7.5-10	21	**0.375**	**4.239**	0.943	4.268	1.005	4.244	1.333	4.252
10-12.5	27	-0.433	4.583	**0.521**	**4.427**	0.722	4.432	0.762	4.607
12.5-15	23	**3.73**	14.67	4.38	**14.34**	4.76	14.37	4.92	14.63
15-17.5	30	-1.76	14.54	-2.43	**14.25**	-1.98	14.26	**-0.945**	14.48
17.5-20	19	-4.75	17.35	-4.82	18.36	-4.72	18.29	**-4.40**	**17.45**
> 20	43	13.01	18.46	**8.46**	**17.59**	6.84	17.56	10.77	18.10
All	193	2.85	14.51	**2.01**	**13.54**	1.81	13,31	2.81	13.63

Equation [5] was assumed to be the best for Sitka spruce. When the mean relative difference (RD) for equation [5] is compared to an existing equation made by Snorrason & Einarsson (2006) it can be seen that equation [5] has less variation and a slightly better fit than the existing equation in diameter classes 1 and 2 (figure 4). The bias and variation is of similar magnitude in the other diameter classes.

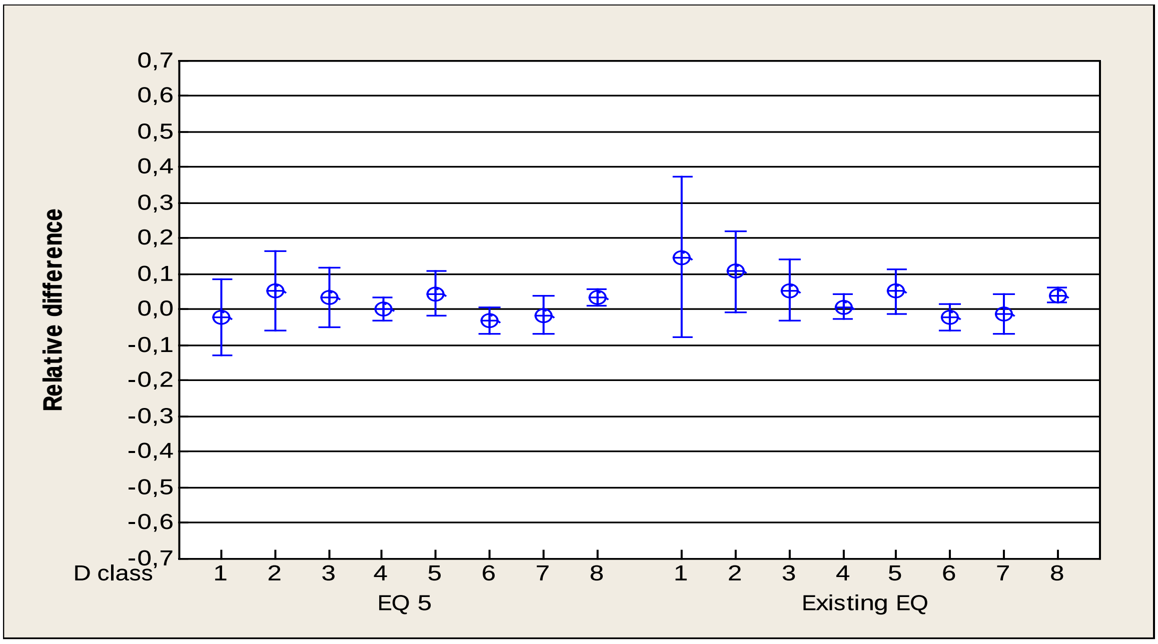


Figure 4. Residuals expressed as relative differences ($(v - \hat{v})/\hat{v}$) between observed and predicted volume for Sitka spruce presented in different diameter (D) classes with 95% confidence intervals. The classes are 1: D <5 cm, 2: <7.5 cm, 3: <10.0 cm, 4: <12.5 cm, 5: <15.0 cm, 6: <17.5 cm 7: <20.0 cm and 8: >20.0 cm. Comparison between equation [5] and the existing equation by Snorrason & Einarsson (2006).

White spruce

The estimated parameters of fitted volume equations tested for white spruce are presented in table 7. Here also, equations [5] and [6] had higher R^2 than equation [4] but the differences are small and all equations have a good fit to the data. Parameter β_4 in equation [6] was not significant, indicating any better fit statistics with an additional diameter variable into equation [5].

Table 7. Parameter estimates and statistics.

Equation	β_0	β_1	β_2	β_3	β_4	R^2
4	-2.1825	1.8464	0.6961			0.9938
5	-2.8593	1.8251	1.8661	-0.9002		0.9942
6	-3.2198	1.7751	1.7563	-0.8137	0.1576	0.9942

All parameters were significant at the 5 % level ($p < 0.05$) except β_4.

Results for validation of bias and RMSE for the equations in different diameter (D) classes and overall fit are presented in table 8. The lowest value in every diameter (D) class is marked bold. Equations [5] perform marginally better than equation [4] and [6] in most of the diameter (D) classes. The value of bias and RMSE for the existing equation tested on this study's data are little greater than equations [4], [5] and [6] (table 7).

Table 8. Mean value of bias (dm^3) and RMSE (dm^3) in different diameter (D) classes and overall fit for equations [4], [5], [6] and existing equation for White spruce. The lowest values of bias and RMSE in different diameter classes are marked bold.

Diameter		Equation 4		Equation 5		Equation 6		Existing equation	
Class	n	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
< 5	10	0.178	0.321	**-0.0034**	**0.2035**	0.0346	0.2489	0.536	0.467
5-7.5	25	**0.109**	1.098	0.215	**1.074**	0.277	1.129	0.509	1.245
7.5-10	16	-0.513	1.867	**0.198**	**1.482**	-0.137	1.824	-0.367	1.760
10-12.5	19	0.749	**2.868**	**0.396**	3.323	0.898	3.10	-1.161	4.129
12.5-15	12	-0.450	7.10	**-0.429**	**6.38**	-0.546	6.64	-3.52	6.55
15-17.5	8	-2.05	10.10	**-1.96**	**10.72**	-2.02	10.67	-4.62	12.56
17.5-20	10	9.27	7.72	7.66	6.92	7.23	6.88	**1.22**	**6.36**
> 20	6	1.69	**24.80**	**-1.09**	25.8	-2.81	26.3	-8.81	28.10
All	106	0.909	7.56	**0.659**	**7.51**	0.566	7.57	-1.224	8.13

In figure 5 the relative difference (RD) for equation [5] is compared to an existing equation made by Snorrason & Einarsson (2006). The existing equation has a local bias in diameter class 1 and looks little crooked compared to equation [5]. Equation [5] has a small local bias in class 7.

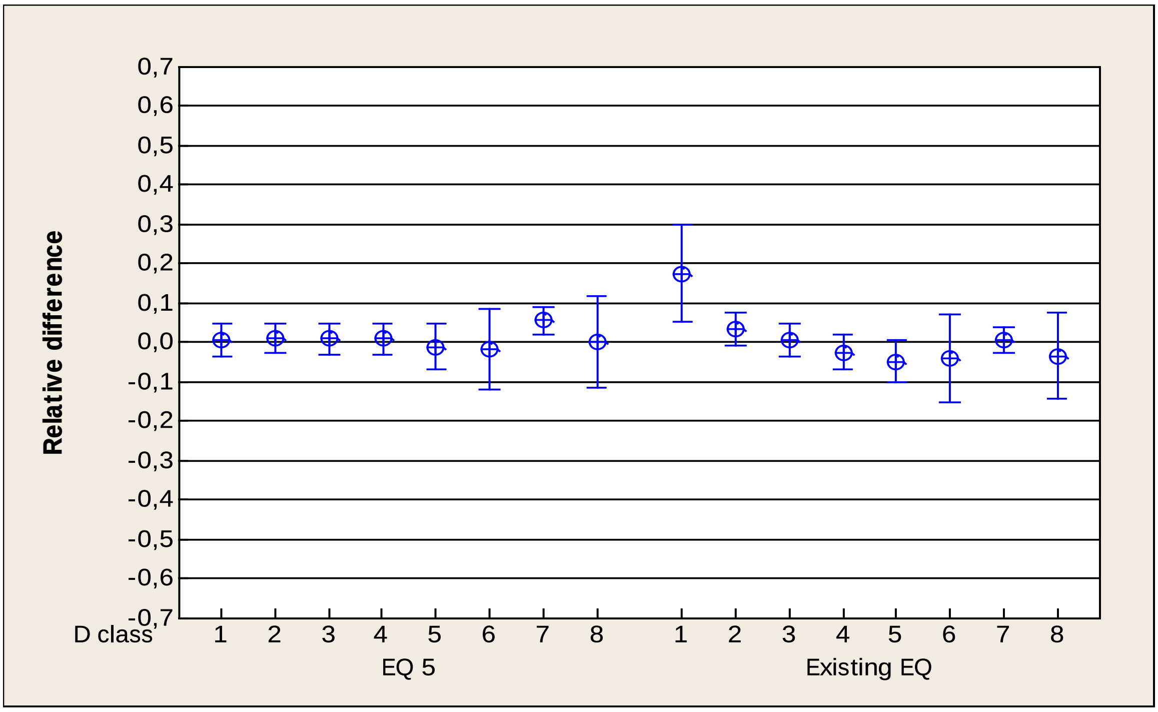


Figure 5. Residuals expressed as relative differences $((v - \hat{v})/\hat{v})$ between observed and predicted volume for White spruce presented in different diameter (D) classes with 95%

confidence intervals. The classes are 1: D <5 cm, 2: <7.5 cm, 3: <10.0 cm, 4: <12.5 cm, 5: <15.0 cm, 6: <17.5 cm 7: <20.0 cm and 8: >20.0 cm. Comparison between equation [5] and the existing equation by Snorrason & Einarsson (2006).

In figures 6, 7 and 8 a graphical comparison between earlier published equations and equation [5] are presented. A previously published study of Brandel (1990), equation 100-01 for Norway spruce in northern Sweden (figure 6) used the same variables as the equation [6] in this study. For the Sitka spruce a previously published study by Bauger (1995) based on samples from plantations on the west coast of Norway is compared (figure 7). Bauger used the same variables as equation [6] in this study, except from that (D+40) is used instead of (D+20). The White spruce was compared to the same study of Brandel as for Norway spruce (figure 8). This was done because no similar equations for comparison with white spruce were found in the literature. The 1 to 1 line is a 100 % fit to equation [5].

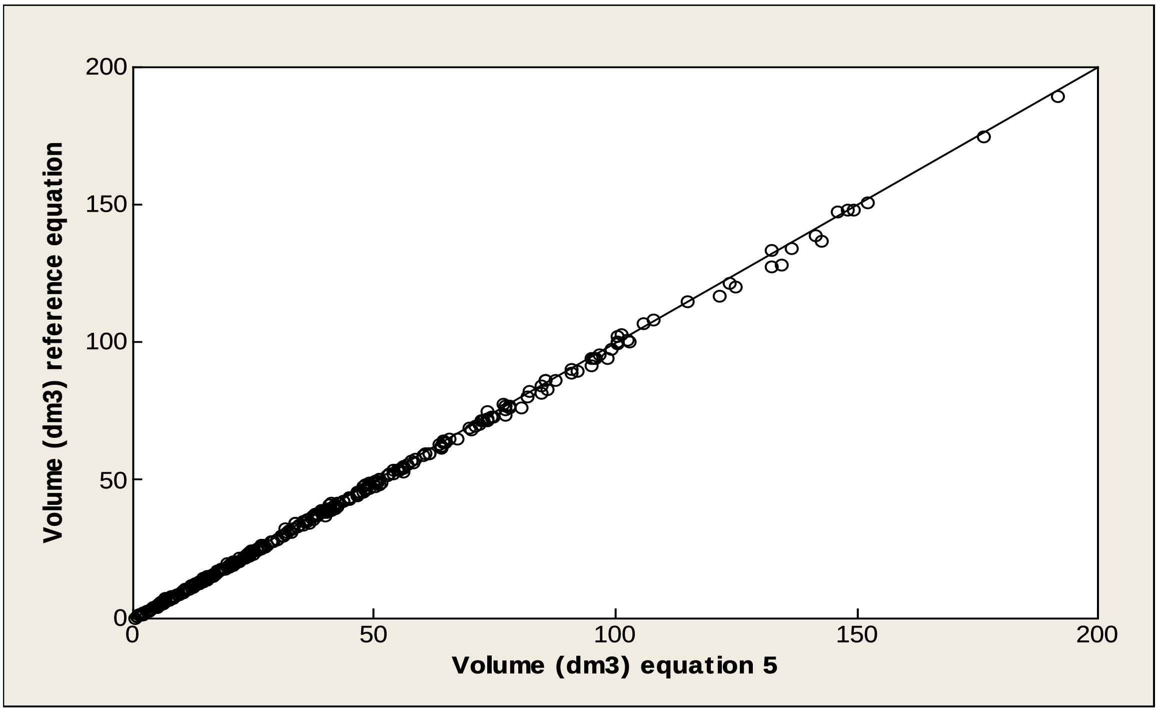


Figure 6. Graphical comparison between the equation published by Brandel (1990) and equation [5] from the present study for Norway spruce.

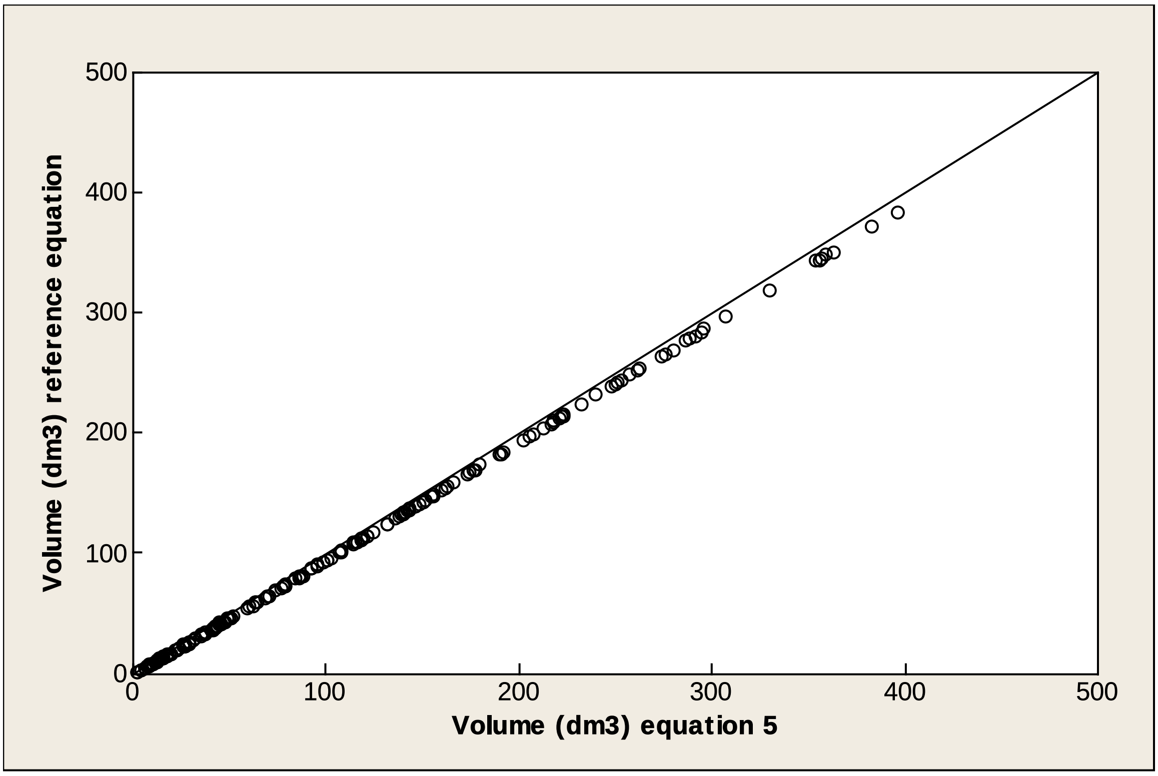


Figure 7. Graphical comparison between the equation published by Bauger (1995) and equation [5] from the present study for Sitka spruce.

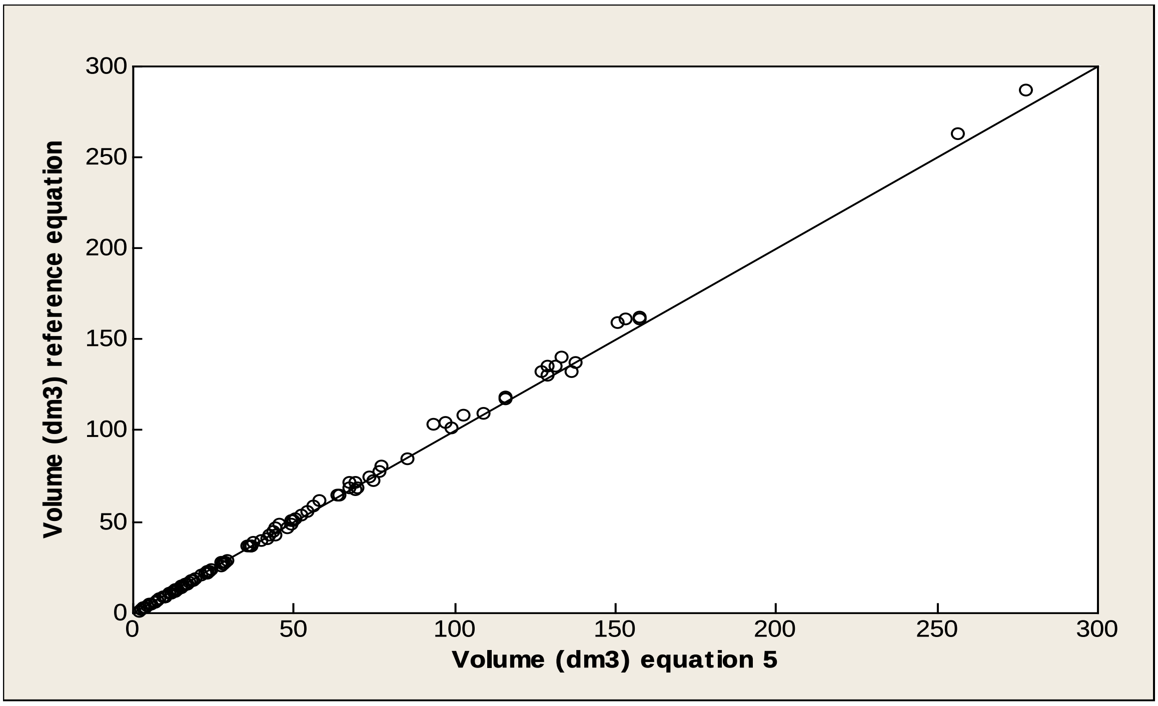


Figure 8. Graphical comparison between the equation published by Brandel (1990) and equation [5] from the present study for White spruce.

The reference studies showed some important differences in predicting the volume of the sample trees compared to equation [5]. The volume prediction was very similar for

Norway spruce, with only a small tendency for underestimation for larger trees with Brandel (figure 6). For Sitka spruce the reference study underestimated the volume of bigger trees (figure 7) and for White spruce the reference study tended to overestimate the volume of bigger trees (Figure 8). This comparison of equation [5] with the reference studies shows clearly the necessity to develop specific volume equations for Icelandic conditions.

Taper equations

Norway spruce

Diameter prediction

The estimated parameters of fitted taper equations for Norway spruce are presented in Table 9. Equation [10] had the highest R^2 and the lowest RMSE, AIC and BIC (table 10). Equations [8] and [9] are almost identical and have similarly fit statistics. Equation [9] had the lowest overall bias.

Table 9. Parameter estimates of the taper equations evaluated in the study.

Equation	8		9		10
β_1	0.9020	β_1	0.8832	*A0*	1.7572
β_2	0.9477	β_2	0.9551	*A1*	-0.3421
β_3	0.1217	β_3	0.1219	*C0*	0.3547
β_4	1.0664	β_4	7.6328	*C1*	-0.2378
β_5	4.3779	β_5	-7.0099	*D0*	0.04598
β_6	3.2313	β_6	-3.9757	*D1*	0.1895
β_7	-2.5545	β_7	4.8417		
β_8	0.04846	β_8	0.2874		
		β_9	6.8226		
$\sigma^2_{\varepsilon t}$	0.3604	$\sigma^2_{\varepsilon t}$	0.3560	$\sigma^2_{\varepsilon t}$	0.3109
$\sigma^2_{\varepsilon r}$	0.000449	$\sigma^2_{\varepsilon r}$	0.000458	$\sigma^2_{\varepsilon r}$	0.000496

Table 10. Fit statistics for Norway spruce in present study.

Equation	R^2	Bias (cm)	RMSE (cm)	AIC	BIC
8	0.9840	-0.01603	0.6334	5755.9	5793.5
9	0.9841	-0.00811	0.6308	5723.9	5765.3
10	0.9858	0.01302	0.5956	5332.3	5362.4

In figure 9 the mean values of bias for equations [9] and [10] are presented by relative height classes. Both equations perform very similarly from bottom up to relative height 0.3. From relative height 0.7 up to the tip, equation [10] displayed a different direction of bias compered to equation [9].

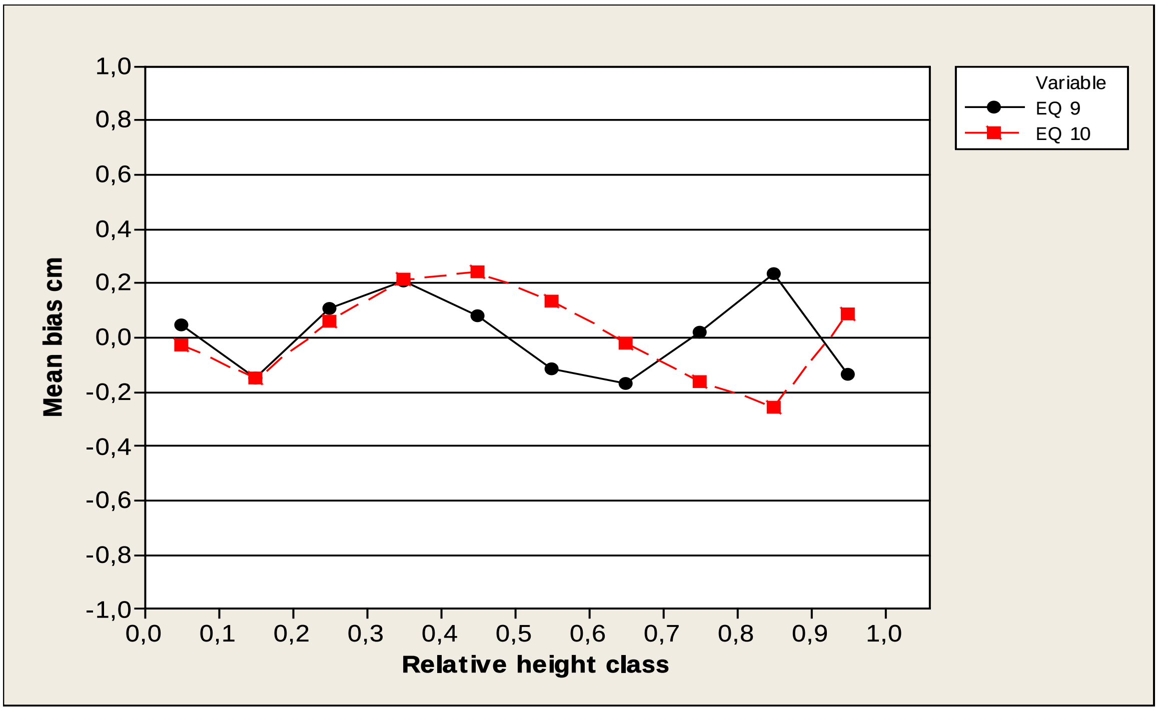


Figure 9. Mean bias in different relative height classes for Norway spruce equations [9] and [10].

Volume prediction

In table 11 the fit statistics regarding volume predictions by the volume equation [5] and the taper equations [8, 9, and 10] on the sample trees are compared. Equation [5] had the highest R^2 and equation [8] had the lowest bias and RMSE. Equation [5] has a larger bias than the other equations, but the RMSE values are very similar among all the equations.

Table 11. Fit statistics for the best volume equation and taper equations predicting volume.

Equation	R^2	Bias (dm^3)	RMSE (dm^3)
5	0.9923	0.5469	4.4770
8	0.9852	0.1086	4.3745
9	0.9844	0.2631	4.4815
10	0.9840	0.1563	4.5385

In figure 10 the relative differences (RD) of the volume prediction for volume equation [5] are compared with the tree taper equations used in the study. Equation [8] and [10] have local bias in predicting volume of trees with diameter at breast height (D) less than 5 centimeters and all the equations have a small local bias in relative height class 7. Equations [5] and [9] performed better in small diameter classes and all equations perform similarly and fairly well in other classes.

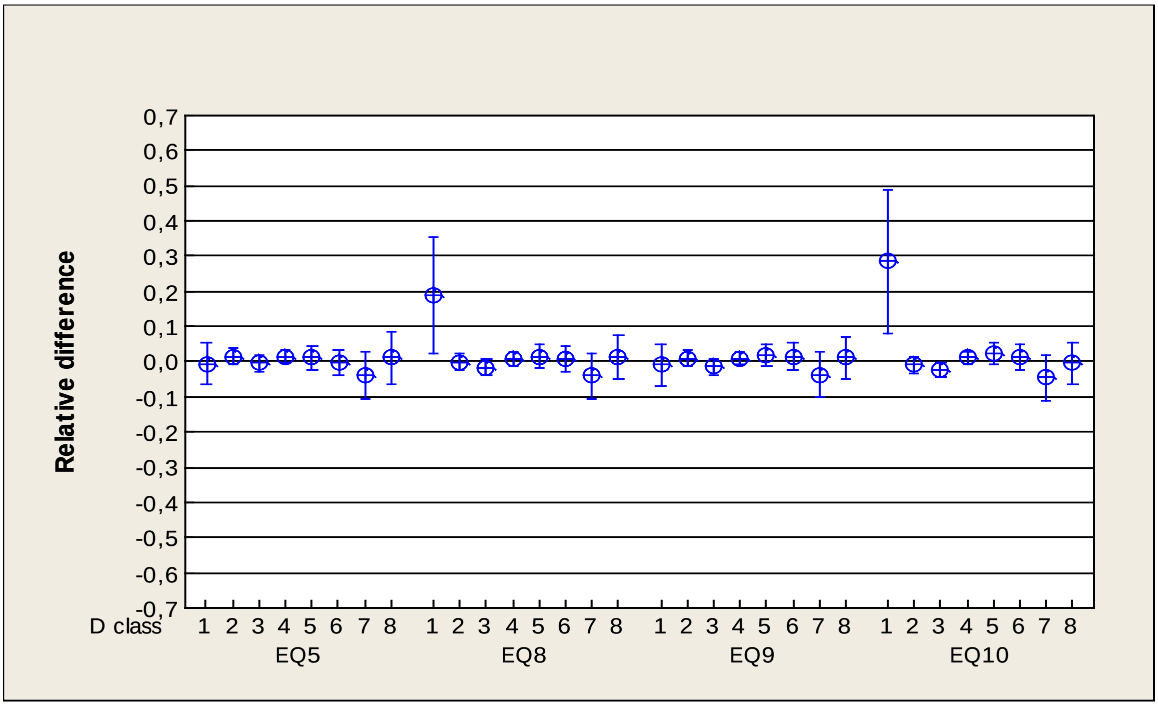


Figure 10. Residuals expressed as relative differences $((v - \hat{v})/\hat{v})$ between observed and predicted volume for Norway spruce presented in different diameter (D) classes with 95% confidence intervals. The best volume equation [5] and taper equations [8, 9, and 10] with 95% confidence intervals. The classes are 1: D <5 cm, 2: <7.5 cm, 3: <10.0 cm, 4: <12.5 cm, 5: <15.0 cm, 6: <17.5 cm 7: <20.0 cm and 8: >20.0 cm.

Sitka spruce

Diameter prediction

The estimated parameters of fitted taper equations tested for Sitka spruce are presented in table 12. In equation [9] parameters β_7 and β_8 were non-signficant so therefore a 7 parameter version of the equation was fitted to the data leading to little higher AIC and little lower BIC values than the 9 parameter version.

Table 12. Parameter estimates of the taper equations evaluated in the study.

Equation	8		9		10
β_1	0.9296	β_1	0.9408	*A0*	1.4579
β_2	0.9845	β_2	0.9814	*A1*	-0.1102
β_3	0.06513	β_3	0.0633	*C0*	0.4033
β_4	0.9527	β_4	8.5617	*C1*	-0.1906
β_5	5.1690	β_5	-4.1980	*D0*	0.1593
β_6	5.7977	β_6	-7.5122	*D1*	0.3801
β_7	-2.8949	β_7	n.s.		
β_8	-0.01628	β_8	n.s.		
		β_9	12.5068		
$\sigma^2_{\varepsilon t}$	0.6715	$\sigma^2_{\varepsilon t}$	0.6696	$\sigma^2_{\varepsilon t}$	0.6576
$\sigma^2_{\varepsilon r}$	0.000469	$\sigma^2_{\varepsilon r}$	0.000478	$\sigma^2_{\varepsilon r}$	0.000521

Equation 10 performed best for all the fit statistics tested for Sitka spruce in diameter prediction (Table 13). Equation [8] and [9] perform very similarly but equation [9] has little lower value for bias and marginally better fit than equation [8] for RMSE, AIC and BIC.

Table 13. Fit statistics for Sitka spruce in present study.

Equation	R^2	Bias (cm)	RMSE (cm)	AIC	BIC
8	0.9861	-0.01388	0.8614	6292.8	6325.4
9	0.9861	-0.00660	0.8610	6286.1	6315.5
10	0.9909	-0.00346	0.8577	6248.6	6274.7

In figure 11 the mean values of bias for equations [9] and [10] are presented by relative height classes. The equations perform very similarly from bottom up to relative height 0.3. From relative height 0.6 up to the tip, equation [10] displayed a different direction of bias compared to equations [9].

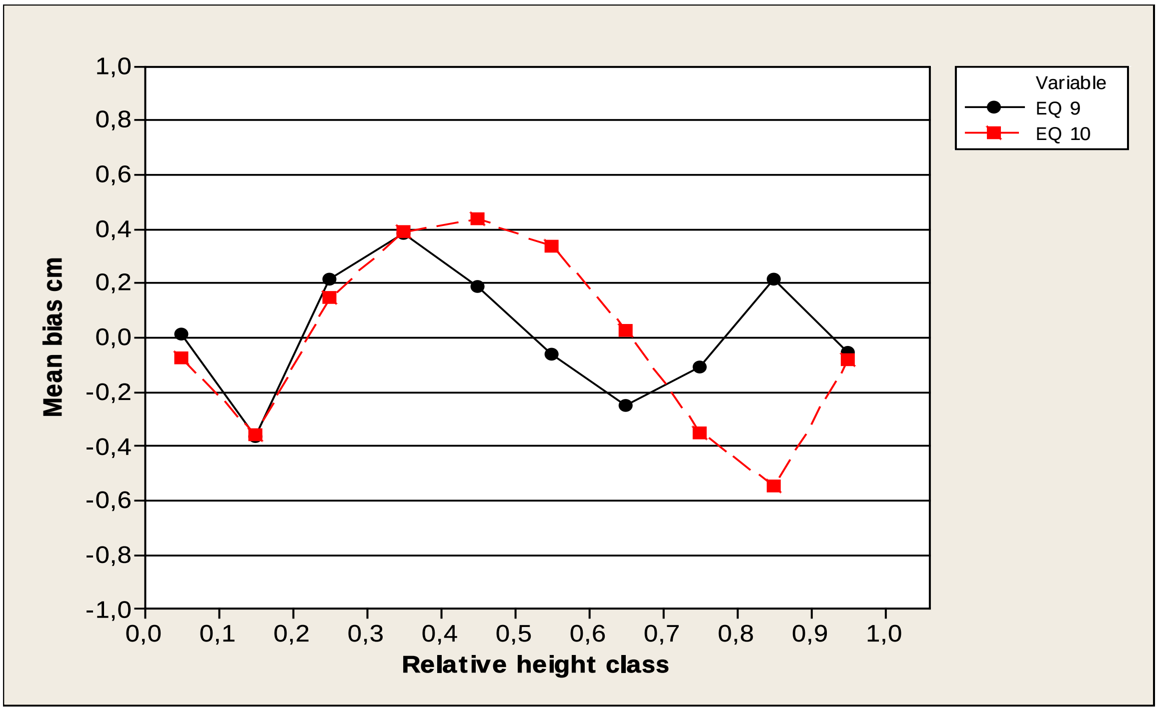


Figure 11. Mean bias in different relative height classes for Sitka spruce equations [9] and [10].

Volume prediction

In table 14 the fit statistics regarding volume predictions by the volume equation [5] and the taper equations [8, 9, and 10] on the sample trees are compared. Equation [10] had the lowest bias and equation [9] the lowest RMSE. Equation [5] had the highest R^2 but also higher bias and the RMSE is marginally higher than the other equations.

Table 14. Fit statistics for taper equations predicting volume.

Equation	R^2	Bias (dm^3)	RMSE (dm^3)
5	0.9892	2.0193	13.5419
8	0.9831	-0.2066	12.9317
9	0.9832	-0.1762	12.9100
10	0.9829	0.1255	13.0216

In figure 12 the relative differences of volume prediction are shown for the best volume equation [5] compared with the three taper equations [8, 9, and 10] tested. The three taper equations perform a little better than equation [5] when bias and RMSE are compared. The taper equations perform very similar in predicting volume for Sitka spruce, except that equation [10] how have the lowest bias have a local bias in predicting volume of trees smaller than 10 cm (figure 12). In the rest of the diameter classes the predictions had a similar trend for all equations.

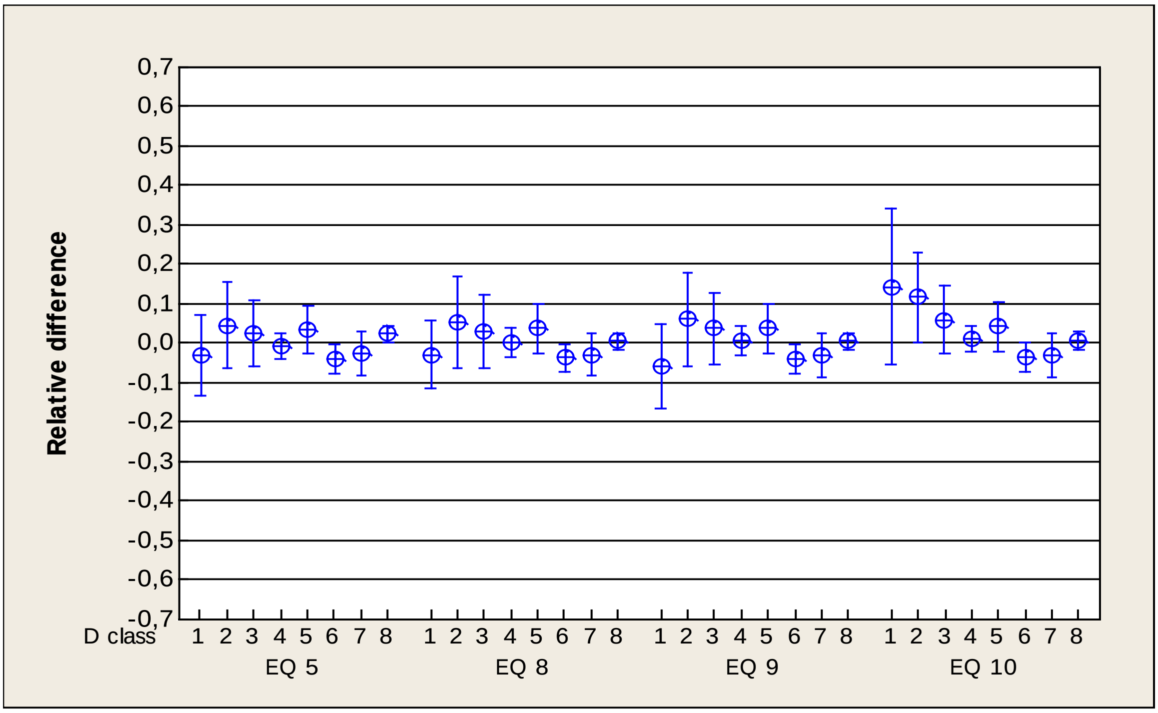


Figure 12. Residuals expressed as relative differences ($(v - \hat{v})/\hat{v}$) between observed and predicted volume for Sitka spruce presented in different diameter (D) classes with 95% confidence intervals. The best volume equation [5] and taper equations [8, 9, and 10]. The classes are 1: D <5 cm, 2: <7.5 cm, 3: <10.0 cm, 4: <12.5 cm, 5: <15.0 cm, 6: <17.5 cm 7: <20.0 cm and 8: >20.0 cm.

White Spruce

Diameter prediction

The estimated parameters of fitted taper equations tested for White spruce are presented in Table 15. In equation [8] the parameter β_8 was not significant and for equation [9] the parameter β_7 was not significant. Therefore a simpler version of the equations was fitted to the data sets leading to slightly lower AIC and BIC values than the original versions.

Table 15. Parameter estimates of the taper equations evaluated in the study.

Equation	8		9		10
β_1	0.9434	β_1	0.9418	$A0$	1.5711
β_2	0.9724	β_2	0.9706	$A1$	-0.1276
β_3	0.06915	β_3	0.07140	$C1$	0.4003
β_4	1.2885	β_4	7.7453	$C2$	-0.1848
β_5	4.5213	β_5	-5.0121	$D1$	0.1038
β_6	4.2600	β_6	-5.0541	$D2$	0.3576
β_7	-3.0399	β_7	n.s.		
β_8	n.s.	β_8	0.05878		
		β_9	9.5035		
$\sigma^2_{\varepsilon t}$	0.4425	$\sigma^2_{\varepsilon t}$	0.4421	$\sigma^2_{\varepsilon t}$	0.3874
$\sigma^2_{\varepsilon r}$	0.000509	$\sigma^2_{\varepsilon r}$	0.000520	$\sigma^2_{\varepsilon r}$	0.000644

Equation [10] had the highest R^2 the lowest bias and RMSE (Table 16). Equations [8] and [9] performed very similarly with values close to each other but equation [9] had a little lower bias.

Table 16. Fit statistics for White spruce in present study.

Equation	R^2	Bias (cm)	RMSE (cm)	AIC	BIC
8	0.9855	-0.01514	0.7035	2378.2	2402.3
9	0.9854	-0.00885	0.7037	2380.0	2406.8
10	0.9865	-0.00503	0.6714	2237.3	2258.7

In figure 13 the mean values of bias for equations [9] and [10] have been plotted to relative height classes. The equations perform very similarly from bottom up to relative height 0.3. From relative height 0.7 up to the tip, equation [10] displayed a different direction of bias compared to equation [9].

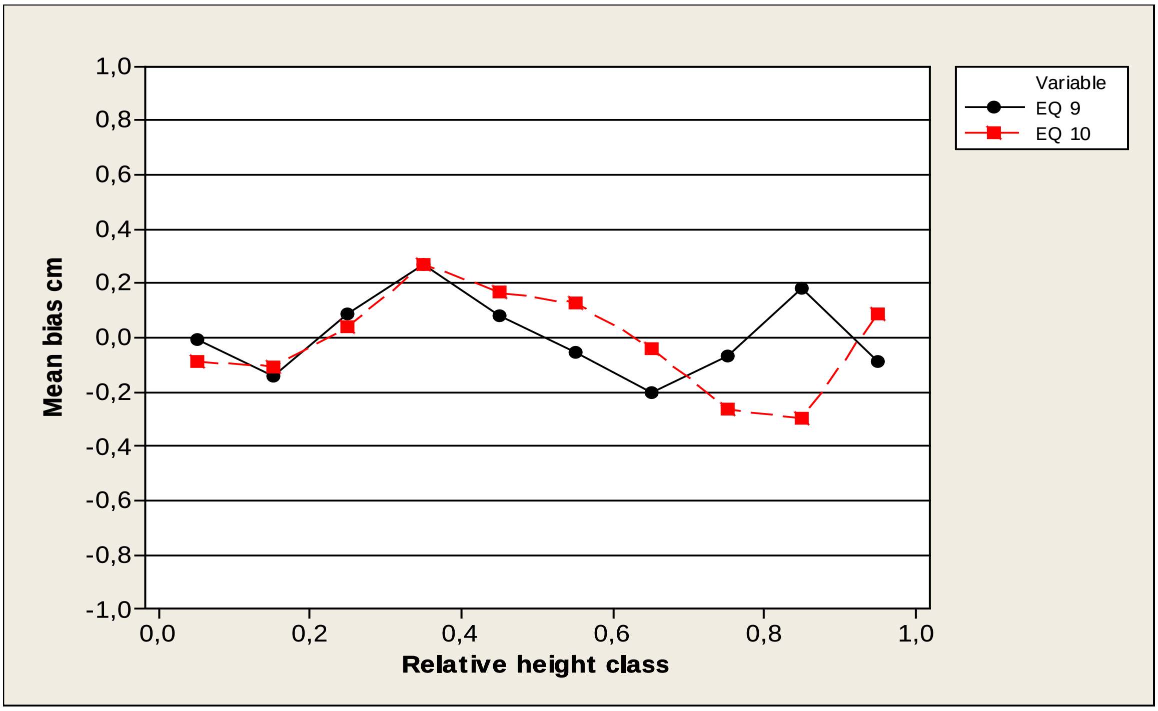


Figure 13. Mean bias in different relative height classes for white spruce, equations [9] and [10].

Volume prediction

In table 17 the fit statistics regarding volume predictions by the volume equation [5] and the taper equations [8, 9, and 10] on the sample trees are compared. The volume equation [5] had the highest R^2 and the lowest bias and RMSE. The values of the fit statistic for the taper equations are very similarly.

Table 17. Fit statistics for taper equations predicting volume.

Equation	R^2	Bias (dm^3)	RMSE (dm^3)
5	0.9942	0.6596	7.5128
8	0.9759	-1.5250	8.1970
9	0.9763	-1.4565	8.1219
10	0.9753	-1.6555	8.2846

In figure 14 the relative differences of the volume prediction for the best volume equation [5] are compared with the three taper equations [8, 9, and 10] tested. Equation [5] has the best overall prediction and equation [9] has the best overall prediction among the taper equations. Equations [5] have the best prediction in all diameter classes except number 7. Equations [8] and [9] perform very similarly but equation [10] shows some under estimation in predicting volume of trees smaller than 5 cm at D. The taper equations showed a similar trend in the rest of the D classes, with a small tendency of over estimation for bigger trees.

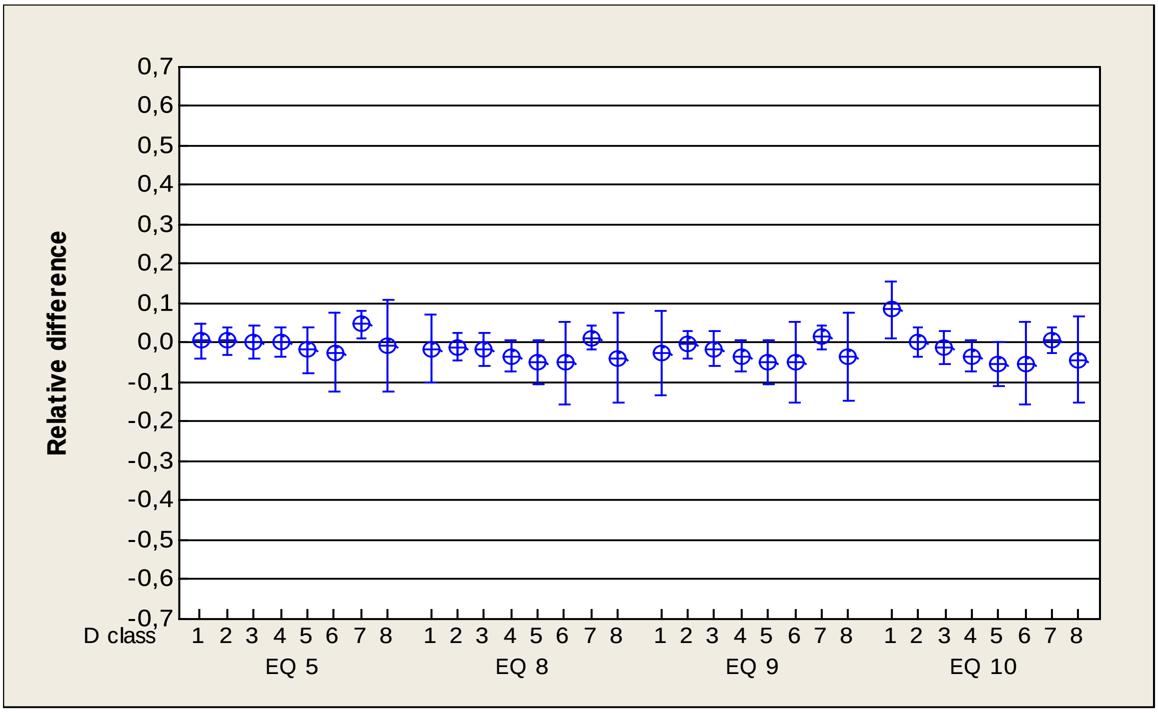


Figure 14. Residuals expressed as relative differences ($(v - \hat{v})/\hat{v}$) between observed and predicted volume for White spruce presented in different diameter (D) classes. The best volume equation is [5] and taper equations [8, 9, and 10] with 95% confidence intervals. The classes are 1: D <5 cm, 2: <7.5 cm, 3: <10.0 cm, 4: <12.5 cm, 5: <15.0 cm, 6: <17.5 cm 7: <20.0 cm and 8: >20.0 cm.

Three stem profiles were simulated for a small, average and a large tree using equation [10] for Sitka spruce as an example (Figure 15, 16). These stem profiles illustrate the changes in stem form along the stem and also differences in stem shape among trees of different size. The relative stem profile predicted from taper equation [10] are more parabolic for small trees (figure 16), consistent with the findings of Forslund (1991) that basal swell and the neolithic proportion at lower stem increased with tree size.

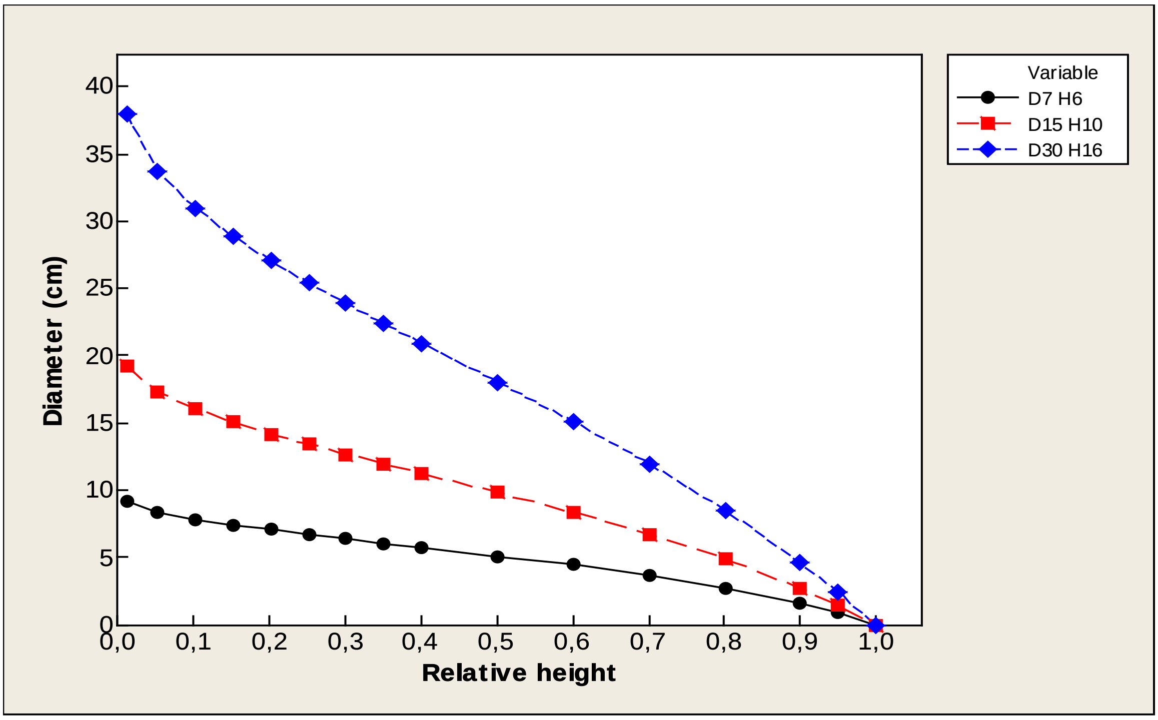


Figure 15. Stem profiles for a small (D=7, H=6), average (D=15, H=10) and large (D=30, H=16) tree derived from equation [10] for Sitka spruce.

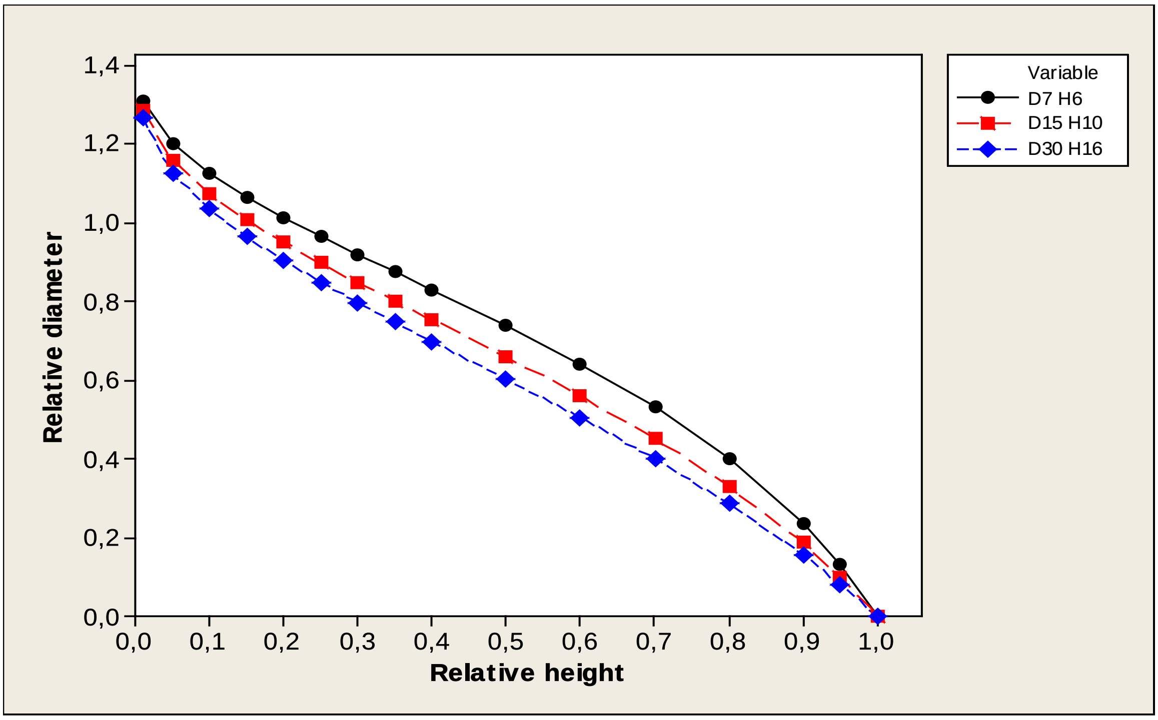


Figure 16. Relative stem profiles for a small (D=7, H=6), average (D=15, H=10) and large (D=30, H=16) tree derived from equation [10] for Sitka spruce.

Discussion

This study evaluates three volume and three taper equations for Norway spruce, Sitka spruce and White spruce. The volume equations predict the total stem volume above stump and the taper equations can be used to predict stem diameter at any height along the tree. The data covers different climate regions around the country, represents different types of stands growing on different soil types and thus covers most of the site conditions suitable for forestry in Iceland. Regarding the sample plots, several sample trees within these were measured and used in this study. That kind of selection should generally be taken into consideration in analysis because trees in the same plot tend to resemble each other more than average and might result in dependence between observations. Parts of the data in the study were obtained from trees that were removed in thinning. The measured trees from these plots may have represented a biased sample since removed trees may have diverged slightly according to stem form from the average for the stand. These sources of potential error could have an effect on the outcome. In table 18 the fit statistics are presented by the origin of the data sets (plot type) used in the study. The volume of Sitka spruce have been calculated using equation [5] and [9] and only plots with more than three sample trees are included. The results do not reveal any significant effects of plot type. The taper equation [9] has marginally lower bias at all plot type except the thinning plot and the RMSE is veary similar for all plot types. The bias and RMSE are higher at the PSP and TP and is explained by bigger sample trees at these sites compared to PtH and PtS.

Table 18. Average bias (dm^3) and RMSE (dm^3) for different data set (plot type) were more than three sample trees selected from a single plot and/or trees removed in thinning. PSP = permanent sample plot, PtH = provenience trail at Hallormsstadur, PtS = provenience trail at Stalpastadir and TP = thinning plots.

PlotType	N	Bias EQ [5]	RMSE EQ [5]	Bias EQ [9]	RMSE EQ [9]
PSP	45	3.82	16.95	2.90	16.56
PtH	50	0.26	4.12	0.16	4.37
PtS	52	2.27	9.44	-1.33	9.44
TP	22	-2.86	21.74	-6.15	20.44

The volume equations were transformed to a logarithmic form, a common procedure to obtain constant variance of the residuals. Equation [5] which had D, H and (H-1.3) as independent variables gave the best results based on fit and validation statistics and was most suitable according to residual analyses for all three species. Equation [4], which includes only D and H as independent variables performed less well and showed a poor performance in predicting volume in smaller diameter classes as well as a large variation in different diameter classes for all species. Adding the forth independent variable (D+20) into equation [6] improved the precision negligible compared to equation [5] and the variable (D+20) was not significant in present data. According to Brandel (1990) the variable (H-1.3) mainly affects trees with breast height diameter lower than 15 cm and the variable (D+20) affects mainly trees with large diameter. The mean diameter for all the species was below 15 cm, which might explain why the variable (D+20) did not improve the volume predictions in present study. Equation [5] also appears to be more precise than existing equations made by Snorrason & Einarsson (2006) for all three species when predictions were analyzed in different diameter classes (Figure 3, 4, 5). The range of tree diameter and height data was similar in that study but the number of sample trees and the number of sites were higher in the present one. Volume equation [5] should therefore be used instead of the existing equations for which sample size was small.

Overall, there was little local bias across relative height classes in the diameter predictions obtained with the three taper equations presented in the study (figures 9, 11 and 13). The RMSE of the taper equation in diameter prediction ranged between 0.59 and 0.87 cm and the value of R^2 ranged between 0.98 to 0.99 for all species. Sitka spruce had the highest RMSE values, ranging between 0.85 to 0.87 cm, whereas Norway spruce had the lowest values, ranging between 0.59 to 0.64 cm. Equation [10] had the best fit statistics for R^2, RMSE, AIC and BIC for all three species tested. It also had the lowest bias for Sitka spruce and white spruce but equation [9] had the lowest bias for Norway spruce. Equations [8] and [9] performed very similarly. Equation [10] gave the best results based on fit and validation statistics, the model simplicity and overall model behavior and is recommended as the stem profile equation in diameter prediction for Norway spruce, Sitka spruce and white spruce in Iceland. The original version of equation [10], which had only three parameters was also tested but did not perform as well as the modified version with six parameters used in this study. Miguel et al. (2011) present similar results where 32 equations gathered from the literature were compared. The result showed that, in general, taper equations with more parameters presented better fittings than simpler equations up to a certain number of parameters (de Miguel et al., 2011). However, in some cases, equations with fewer parameters performed better than certain equations with more parameters.

The original version of equation [10] was developed and tested on six conifer species of Northern California (Biging, 1984). Only fit statistics for ponderosa pine (*Pinus ponderosa*), and white fir (*Abies concolor* (Gord. & Glend.) Lindl. (Iowiana [Gord.])), were presented in the study. For ponderosa pine the R^2 value was 0.990 and for white fir 0.989, which are of similar magnitude as the modified version used in this study where the values are 0.985 for Norway spruce, 0.990 for Sitka spruce and 0.986 for white spruce. The RMSE values for ponderosa pine and white fir were presented at different relative heights in the reference study and were a little higher compared with this study at all relative heights except in the section closest to the ground were the values were similar. Bias at all relative heights was of similar magnitude, marginally higher in the study of Biging.

The performance of the taper equations was almost identical from the bottom up to a relative height 0.4, with similar value of bias for all species tested (Figure 9, 11, 13). From relative height 0.4 up to the tip, equation [10] showed different bias compared to equations [8] and [9], which were almost identical up to the tip for all species. The largest mean bias for all equations and all species was at relative height 0.4 up to 0.6 and at relative height 0.8 up to 0.9.

The performance of volume prediction based on fit statistics showed a slightly different trend than the diameter prediction along the stem. Equation [10], which gave the best results in diameter prediction for all species, did not perform as well in volume prediction of small trees with diameter at breast height (D) less than 5 cm. For Norway spruce, equation [8] had the best fit statistic in volume prediction but when the relative differences were examined visually in different diameter classes (Figure 10) the same was noticed as for equation [10] in predicting volume of small trees. The fit of all the equations was similar in the other diameter classes for Norway spruce. For Sitka spruce, equation [5] had the highest R^2 value (0.989) but also the highest bias and RMSE. Equation [10] had the lowest bias and equation [9] the lowest RMSE. When the relative differences in diameter classes among the taper equations were examined visually (Figure 12) it revealed the same trend for equations [5], [8] and [9] but equation [10] had a different trend in volume prediction of small trees and was more biased in diameter classes 1, 2 and 3. For white spruce, equation [5] had the best fit statistic and only small local bias in diameter class 7 (Figure 14). The taper equations had a similar trend in different diameter classes but were more biased. The volume estimation of small trees was better for equations [8] and [9] than for equation [10] and with increasing stem volume all the taper equations tended to over predict the volume of white spruce except in diameter class 7.

None of the taper models performed very well in volume prediction. Equation [10] is clearly biased in predicting volume of small trees and the same was true for equation [8] for Norway spruce. Equations [5] and [9] seem to be more flexible in predicting the volume of small trees as well as other tree sizes. So the best choice in volume prediction among the taper equations is equation [9]. However, more independent data to judge the equations rehabilitee might be desired.

Recently, Icelandic Forest Service introduced a new forest management planning system. The new sets of taper equations are an important component of that system because the amount of timber in the forests can be evaluated and the forest resources used more effectively. In coming years when trees become bigger than the sample trees used in this study a further development of the volume and taper equations will be needed. Also additional data should be collected from areas that are not presented in this study.

Acknowledgements

I would like to thank my supervisor Kenneth Nyström at the SLU for great guidance and help along the way. I would like to express my gratitude to Þröstur Eysteinsson at the Icelandic forest service for his comments on my study. Finally I would like to thank my family for their support during this thesis and Margaret Anne Johnson for reading the manuscript over. Thanks.

Bibliography

Aradottir, A., & Eysteinsson, T. (2005). *Restoration of birch woodlands in Iceland. Restoration of boreal and temperate forests.* (J. A. Stanturf, & P. Madsen, Eds.)

Assmann, E. (1970). *The Principles of Forest Yield Study.* New York: Pergamon Press.

Avery, T. E., & Burkhart, H. E. (2002). *Forest measurements.* New York: Mcgraw-Hill, 5th edn.

Baskerville, G. L. (1972). Use of logarithmic regression in the estimation of plantbiomass. *Can. J. For. Res., 2*, 49-53.

Bauger, E. (1995). *Funksjoner og tabeller for kubering av stående trær. Furu, gran og sitkagren på Vestlandet. [Tree volume functions and tables. Scots pine, Norway spruce and Sitka spruce in western Norway] In Norwegian with english summary.* Norsk institutt for Skogforskning og institutt for Skogfag NLH, Ås.

Bi, H. (2000). Trigonometric variable-form taper equations for Australian eucalypts. *Forest Sci., 46*(3), 397-409.

Biging, G. S. (1984). Taper equations for second-growth mixed conifers in northern California. *Forest Sci., 30*(4), 1103-1117.

Bjarnadottir, B., Inghammar, A. C., Brinker, M. M., & Sigurdsson, B. D. (2007). Single tree biomass and volume functions for young Siberian larch trees (Larix sibirica) in eastern Iceland. *Icel. Agric. Sci, 20*, 125-135.

Blöndal, S. (1993). Socioeconomic Importance of forests in Iceland. In J. Alden, J. L. Mastrantonio, & S. Ødum (Ed.), *Forest development in cold climates. NATO ASI Series. Series A: Life Sciences*, *244*, pp. 1-13.

Blöndal, S. (2007). Innfluttu skógartrén iv. Rauðgreni (Picea abies (L.) Cast.). (In Icelandic). *Skógræktarritið, 1*, 20-23.

Brandel, G. (1990). *Volymefunktioner för enskilda trad.* Department of forest yield research. SLU.

Bruce, D., Curtis, R. O., & Vancoevering, C. (1968). Development of a system of taper and volume tables for red alder. *For. Sci., 14*, 339-350.

Burkhart, H. E., & Gregoire, T. G. (1994). *Handbook of statistics, vol. 12, chap. Forest biometrics, pp377-407. New York: Elsevier Science Publisher.* (Vol. 12). New York: Elsevier Science Publisher.

Burnham, K. P., & Anderson, D. R. (1998). *Model selection and inference: a practical information-theoretic approach.* Berlin Heidelberg New York: Springer.

Cao, Q. V., Burkhart, H. E., & Max, T. A. (1980). Evaluation of two methods for cubic-volume prediction of Loblolly pine to any merchantable limit. *Forest Sci., 26*(1), 71-80.

Clutter, J. L., Fortson, L. V., Pienaar, G. H., Brister, G. H., & Bailey, R. (1983). *Timber management: A quantitative approach.* Malabar, Florida: Krieger publishing company.

Courbet, F., & Houllier, F. (2002). Modelling the profile and internal structure of tree stem. Application to Cedrus atlantica (Manetti). *Ann. For. Sci, 59*, 63-80.

Demaerschalk, J. P. (1972). Converting volume equations to compatible taper equations. *Forest Sci., 18*(3), 241-245.

Demaerschalk, J. P., & Kozak, A. (1977). The whole-bole system: a conditioned dual equation system for precise prediction of tree profiles. *Can. J. For. Res., 7*, 488-497.

de-Miguel, S., Shater, Z., Kraid, B., Mehtätalo, L., & Pukkala, T. (n.d.). Modelling the taper of Pinus brutia in Syria. *Manuscript submitted to European Journal of Forest research.*

Diéguez-Aranda, U., Castedo-Dorado, F., Álvarez-González, J. G., & Rojo, A. (2006). Compatible taper function for Scots pine plantations in northwestern Spain. *Can. J. For. Res., 36*, 1190-1205.

Eerikainen, K. (2001). Stem volume models with random coefficients for Pinus kesiya in Tanzania, Zambia and Zimbabwe. *Can. J. For. Res., 31*, 879-888.

Eriksson, H. (1973). Volymfunktioner för stående träd av ask, asp, klibbal och contorta - tall. Summary. [Tree volume functions for ash, aspen, alder and lodgepole pine in Sweden]. (Fraxinus excelsior L., Populus tremula L., Alnus glutinosa (L.) Gartn., Pinus contorta Dougl. *26*, 26.

Eysteinsson, T. (2009). *Forestry in a treeless land.* Lustgarden.

Fabian, U. C., & Oliver, W. W. (2006). Individual tree height increment model for managed even-aged stands of ponderosa pine throughout the western United States using linear mixed effects models. *Forest Ecologi and Management, 221*, 147-154.

Fang, Z., & Bailey, R. (2001). Nonlinear mixed-effects modeling for slash pine dominant height growth following intensive silviculture treatments. *Forest Science, 47*, 287-300.

Finney, D. J. (1941). On the Distribution of a variate whose logarithm is normally distributed. *Journal of the Royal Statistical Society, Supp B7, 7(2)*, 155-161.

Forslund, R. R. (1991). The power function as a simple stem profile examination tool. *Can. J. For. Res., 21*, 193-198.

Freysteinsson, G. (1995). *Freysteinsson, G. 1995. Boniteringskurver for sitkagran)Picea sitchensis (Bong.) Carr.) på sörvest-Island. [Site index curves for sitka spruce (Bong.) Carr.) in south west Iceland]. (In Norwegian with English summary).* Institut for Skogfag.

Garber, S. M., & Maguire, D. A. (2002). Modeling stem taper of three central Oregon species using nonlinear mixed effects models and autoregressive error structures. *Forest Ecologi and Management, 179*, 507-522.

Goulding, C. J., & Murray, J. C. (n.d.). Polynomial taper equations that are compatible with tree volume equations. *N. Z. J. For. Sci., 5*, 313-322.

Gray, H. R. (1956). The Form and Taper of Forest-Tree Stems. Imperial Forestry Institute.

Harris, A. S. (1984). *Sitka spruce. An American wood.* Department of Agriculture, Forest Service, Pacific Northwest Forest and Range Experiment Station. U.S. Forest service.

Heidarsson, L., & Pukkala, T. (2011). Taper functions for lodgepole pine (Pinus contorta) and Siberian larch (Larix sibirica) in Iceland. *Icel. Agric. Sci., 24*, 3-11.

Hjelm, B. (2011). Taper and volume equations for poplar trees growing on farmland in Sweden. *Swedish university of agricultural sciences. Department of energy and technology.*

Husch, B., Miller, C. I., & Beers, T. W. (1972). *Forest mensuration.* New York: Ronald Press Company.

Jonson, T. (1928). Några nya metoder för beräkning av stamvolym och tillväxt hos stående träd. Summary: Some new methods for calculating volume and increment of standing timber. In Swedish. *SST, (26) 3-4*, 423-427.

Jordan, L., Berenhaut, K., Souter, R., & Daniels, R. F. (2005). Parsimonious and completely compatible taper, total and merchantable volume models. *Forest science, 51*(6), 578-584.

Karlsson, K. (2005). Growth allocation and stand structure in Norway spruce stands - Expected taper and diameter distribution in stands subjected to different thinning regimes. Doctoral diss. Dept of Bioenergy, SLU, Uppsala, Sweden. *Acta Universitatis Agriculturae Sueciae, 75*, 1-35.

Korhonen, K. T., & Eerikainen, K. (2001). Tilavuusmallit [In Finnish]. (M. Maltamo, & S. Laukkanen, Eds.) *Silva Carelica, 36*(Metsää kuvaavat mallit), 84-91.

Kozak, A. (1988). A variable-exponent taper equation. *Can. J. For. Res., 18*, 1363-1368.

Kozak, A. (1997). Effects of multicollinearity and autocorrelation on the variable-exponent taper functions. *Can. J. For. Res., 27*, 619-629.

Kozak, A. (2004). My last words on taper equations. *The Forestry chronicle, 80*(4), 507-515.

Kozak, A., Munro, D. D., & Smith, J. H. (1969). Taper functions and their application in forest inventory. *For. Chron., 45*(4), 278-283.

Kristinsson, H. (1995). Post-settlement history of Icelandic forests. *Icel. Agric. Sci., 9*, 31-35.

Kublin, E., Helene, A., & Lappi, J. (2008). A flexible regression model for diameter prediction. *European Journal of Forest Research, 127*, 415-428.

Kurinobu, S., Prehatin, D., Mohanmad, N., & Matsune, K. (2007). A stem taper equation compatible to volume equation for Paraserianthes falcataria in Pare, East Java, Indonesia: its implication for the plantation management. *J. For. Res., 12*, 473-478.

Laar, A. V., & Akça, A. (2007). *Forest mensuration.* Dordrecht, Netherlands: Springer.

Laasasenaho, J. (1982). Taper curve and volume functions for pine, spruce and birch. *Communicationes Instituti Forestalis Fenniae, 108.*

Lappi, J. (1986). Mixed linear models for analyzing and predicting stem form variation of scots pine. *Communicationes instituti forestalis fenniae, 134*, 1-69.

Lappi, J. (2006). A multivariate, nonparametric stem-curve prediction method. *Can. J. For. Res., 36*, 1017-1027.

Larson, P. R. (1963). Stem Form Development of Forest Trees. *Forest Science, Monograph 5.*

Lindström, M. J., & Bates, D. M. (1990). Nonlinear mixed-effects models for repeated measures data. *Biometrics, 46*, 673-687.

Ljunglof, J. (2010). *Mall word 2007.*

Loetsch, F., & Haller, K. E. (1973). *Forest Inventory. Vol. I.* München: BLV Verlagsgesellschaft mbH.

Loftsson, J. (1993). Forest development in Iceland. In J. Alden, J. L. Mastrantonio, & S. Ödum (Ed.), *Forest development in cold climates* (pp. 453-461). NATO ASI Series. Series A: Life Sciences.

Matney, T. G., & Schultz, E. B. (n.d.). Mesavage and girard form class taper functions derived from profile equations. *Proceedings of the 15th central hardwood forest conference*, (pp. 77-85).

Max, T. A., & Burkhart, H. E. (1976). Segmented polynomial regression applied to taper equations. *Forest Sci., 22*, 283-289.

Muhairwe, C. K. (1999). Taper equations for Eucalyptus pilularis and Eucalyptus grandis for the north coast in New South Wales, Australia. *Forest Ecology and Management, 113*, 251-269.

Näslund, M. (1940). Funktioner och tabeller för kubering av stående träd. Tall, gran och björk i norra Sverige. Zusammenfassung: Funktionen und tabellen zur kubierung stehender bäume. Kiefer, fichte und birke in nordschweden. (In Swedish with German summary). *Meddelanden från Statens Skogforskningsinstitutet, 32*(4), 87-132.

Näslund, M. (1947). Funktioner och tabeller för kubering av stående träd. Tall, gran och björk í Södra Sverige samt i hela landet. Summary: Functions and tables for computing the cubic volume of standing trees. Pine, spruce and birch in southern

Sweden, and in the whole of S. *Meddelanden Från Statens skogsforskningsinstitut, 36*(3).
Newnham, R. M. (1988). *A variable-form taper equation.* Petawawa National Forest Institute. Petawawa: Canadian Forest Service.
Newnham, R. M. (1992). Variable form taper functions for four Alberta tree species. *Can. J. For. Res, 22*, 210-223.
Norrby, M. (1990). *Volym- och formtalsfunktioner för Larix sukaczewii och Larix sibirica på Island. [Volume and form factor functions for Larix sukaczewii and Larix sibirica in Iceland]. (In Swedish with English summary).* Institutionen för skogsskötsel. Umeå: Sveriges Lantbruksuniversitet.
Ormerod, D. W. (1973). A simple bole model. *For. Chron., 49*, 136-138.
Pardé, J. (1980). Forest biomass. *Forestry Abstracts, 41*, 343-362.
Parresol, B. R., Hotvedt, J. E., & Cao, Q. V. (1987). A volume and taper prediction system for bald cypress. *Can. J. For.Res., 17*, 250-259.
Philip, M. S. (1994). *Measuring tree and forests* (2nd ed.). Wallingford, UK: CAB International.
Pinherio, J. C., & Bates, D. M. (2000). *Mixed-Effects Models in S and S-Plus.* New York: Springer-Verlag.
Rojo, A., Perales, X., Sanchez-Rodriguez, F., Alvarez-Gonzalez, J. G., & von Gadow, K. (2005). Stem taper functions for maritime pine (Pinus pinaster Ait.) in Galicia (northwestern Spain). *Eur.J. For. Res., 25*, 177-186.
Rojo, A., Perales, X., Sánches-Rodrígues, F., Álvarez-González, J. G., & von Gadow, K. (2005). Stem taper functions for maritime pine (Pinus pinaster Ait.) in Galicia (Northwestern Spain). *Eur. J. Forest Res., 124*, 177-186.
Schumacher, F. X., & Hall, F. S. (1933). Logarithmic expression of timber-tree volume. *Journal of Agricultural Research, 47*, 719-734.
Searle, S. R. (1987). *Linear models for unbalanced data.* New York: John Wiley & Sons, Inc.
Snorrason, A. (2014). *Information collected by the Icelandic NFI and reported in the Joint Forest Europe/UNECE/FAO Questionnaire on Pan-European Indicatiors for Sustainable Forest Mangement.* Geneva: UNECE/FAO Forestry and Timber Section.
Snorrason, A., & Einarsson, S. F. (2006). Single-tree biomass and stem volume functions for eleven tree spices used in Icelandic forestry. *Icel. Agric. Sci., 19*, 15-24.
Snorrason, A., Kjartansson, B. Þ., Gunnarsson, E., & Eysteinsson, T. (2005). *Global Forest Resources Assessment Update 2005. Iceland. Country Report.* FAO.
Sprugel, D. G. (1983). Correcting for bias in log-transformed allometric equations. *Ecology, 64*, 209-210.
Spurr, S. H. (1952). *Forest inventory.* New York: The Ronald Press Co.
Sterba, H. (1980). Stem curves – a review of the literature. *Forestry Abstracts, 41*(4), 141-145.
Tarp-Johansen, M. J., Skovsgaard, J. P., Madsen, S. F., Johannsen, V. K., & Skovgaard, I. (1997). Compatible stem taper and stem volume functions for oak (Quercus robur L and Q petraea (Matt) Liebl) in Denmark. *Ann. Sci. For., 54*, 577-595.
Tasissa, G., & Burkhart, H. E. (1998). An application of mixed effects analysis to modeling thinning effects on stem profile of loblolly pine. *For. Ecol. Manage., 103*, 87-101.
Thomas, C. E., & Parresol, B. (1991). Simple, flexible, trigonometric taper equations. *Can. J. For. Res., 21*(7), 1132-1137.

Thomas, C. E., & Parresol, B. R. (1991). Simple, flexible, trigonometric taper equations. *Can. J. For. Res., 21*, 1132-1137.
Trincado, G., & Burkhart, H. (2006). A generalized approach for modelling and localizing stem profile curves. *Forest Science, 52*, 670-682.
Valentine, H., & Gregoire, T. (2001). A switching model of bole taper. *Can. J. For. Res., 31*, 1400-1409.
Vonesh, E. F., & Chinchilli, V. M. (1997). *Linear and nonlinear models for the analyses of repeated measurements.* New York: Marcel Dekker.