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Department of Economics

# Barking up the right tree: binomial option valuation of investments in steel making

A case study of Metals and Polymers (Ukraine)

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Abstract

This paper investigates the impact of real options on the value of a particular business entity. Theoretically, the paper benefits from use of an option pricing model which is based on a two-state (binomial) framework. This model provides an intuitive appeal by visually showing the evolution of the value of a business in terms of discrete time intervals. In contrast to majority of similar studies, this paper utilizes an extended version of the binomial framework that combines both volatility and drift (in actual expression, it is substituted by risk-free rate).

Empirically, the analysis is supported by financial data from Ukrainian steel-making company *Metals and Polymers Ltd*. Properties of production process, as well as institutional environment, enable to detect two business opportunities examined by means of the theoretical model selected. These opportunities are option to expand and option to abandon. The potential of the company in terms of profitability is estimated using conventional NPV analysis, with no options included. This estimation is then enhanced by implementing the options chosen.

To account for different economic scenarios, some independent variables are taken fixed while others are loosen to float in the imposed intervals. Among the variables, changeable (floating) ones are rate of corporate tax, rate of volatility and discount rate. It is shown that in the presence of options the value of the company is significantly higher than without them. Moreover, its value is subject to noticeable fluctuations depending on alterations in the changeable variables.

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# 1. Introduction

## 1.1 Problem background

At the beginning of the 2000s Ukraine underwent economic boom that spurred the demand for a high-quality steel with an anticorrosive cover used mostly in production of the building materials – roof tiles, sandwich panels – and in the electronics. Briefly, the anticorrosive protection that increases the resistance of steel to the detrimental impact of water, sun and temperature oscillations, is provided by two elements coating the "naked" steel – zinc and polymeric paint.

Table 1 shows that in the period 2006-2011 consumption of the protected types of steel in Ukraine almost doubled. This provoked active investments in the production capacities. By now, in the country there function five plants which are capable of producing about 750 000 tons of galvanized (zinc-coated) steel and 145 000 tons of polymerized steel (polymeric paint is laid over the zinc coating).

**Table 1. Consumption of galvanized and polymerized steel in Ukraine (in tons)**

| Production type | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|---|
| Galvanized steel | 180,000 | 258,000 | 345,000 | 250,000 | 349,000 | 365,000 |
| Polymerized steel | 98,000 | 140,000 | 180,600 | 140,000 | 167,000 | 210,000 |

**Source:** Ilyin, 2010; Dorozjovets, 2012

In future, a few new projects are planned that include expanding current capacities, as well as developing new production lines. There is a list of the companies, which investments are most substantial. For instance, *Metals and Polymers* aims to expand the lines for galvanization and polymerization, and install the equipment set to churn out the cold-rolled steel, which can be sold in the market, as well as used as an input element in the galvanization process (www, Bronx International, 2012). *Unisteel* is going to expand its storage capacities to optimize the distribution timing of the produced steel (Ilyin, 2010). *Ilyich Steel & Iron Works* is expected to invest heavily in the modernization of its galvanization line after the company was acquired by the more powerful rival (Ilyin, 2011). Current capacities of these companies are given in Table 2.

In Ukraine, the drivers that positively contribute to the development of the industry are traditionally combined with the factors that threaten or decelerate investments. On the growth side, there is an internal demand associated with a need to thoroughly restore obsolete fixed capital owned by both industrial and civil public sectors. By some estimates, there are industries in the country where the fixed capital is wore up to 80% that implies a huge market for the production of steel industry, including protected steel (www, State Statistics Committee of Ukraine, 2012). Another growth-inducing factor comes from the rising purchasing power of the households that managed to increase their real incomes tenfold in the last decade. But still, their consumption of protected steel is significantly lower than in Europe or U.S. This fact allows steel producers to count on the potential interest of individual families (Ilyin, 2009).

**Table 2. Production capacities of galvanized and polymerized steel in Ukraine**

| Company | Galvanized steel | Polymerized steel |
|---|---:|---:|
| Metals and Polymers | 100,000 | 75,000 |
| Unisteel | 100,000 | - |
| AZST-Colour (Ruukki Ukraine) | - | 70,000 |
| Ilyich Steel & Iron Works | 350,000 | - |
| Modul | 200,000 | - |
| **Total** | **750,000** | **145,000** |

**Source:** Ilyin, 2011; Ilyin, 2009. The data given in tons

Main risks that jeopardize the realization of the investment projects in steel industry in Ukraine could be boiled down to the volatility in prices for inputs and outputs and the imperfect institutional framework. Muharam (2010) points out that in terms of price fluctuations steel producers appear to be in an unique situation as both prices for inputs and prices for ready products follow a random walk. In the sector there is also a phenomenon of cost-price squeeze that assumes that the inflation in input prices may outpace the growth in prices for products and that may lead to forced divestment (Kotas, 2011). Ukrainian steel producers, including those making protected steel, are importing and exporting heavily and therefore increasingly subject to the changes in the world market conjuncture. Disadvantages of the institutional framework in Ukraine include problems of leading entrepreneurial activities, the absence of impartial and independent court system, the threat of being a victim of unfriendly acquisition, the risk of state interference and so forth (European Business Association, 2007).

In view of the above mentioned, the role of financial models to capture and evaluate major uncertainties before the investment project is initiated becomes very significant. As practice shows, theory of real options has a potential to take into account future changes in key variables that gives investors a flexibility in making decisions (Brealey and Myers, 2003). In particular, the theory is equipped with tools that allow tracking alterations in prices and discount rates, modeling the evolution of demand and, as a result, suggesting the right time to expand, delay, mothball or abandon the projected investments. In the course of analysis these opportunities are detected to maximize worth or minimize potential losses of an investor. It means either active development in the time of bright prospects through expansion and takeover, or phasing out operations following the economic slowdown (*ibid*).

This thesis presents a case study based on the data from the Ukrainian company *Metals and Polymers Ltd.* that specializes in production of galvanized steel and galvanized steel with polymeric coating (polymerized steel). Initially, long before the current capacities were put into work, the company was considered to be a multi-stage industrial project with the opportunity to expand dictating the choice of the size of the location site (Pers.Com., Risukhin, 2012). In terms of the real options analysis (ROA), this opportunity is regarded as an option to expand and could be evaluated using the algebraic derivations of the binomial option pricing model. Simultaneously, to reflect the opposite side of the project associated with the risk of loss, the opportunity to discard the operations by selling them to a willing buyer should be analysed. Again, in the academic terminology, the opportunity to leave a business implies an option to abandon. This option protects investors from the excessive losses to be incurred should the business project in place kept alive. The detailing of the theoretical concepts behind ROA, the choice of method, as well as the financial information of the company under consideration will be given in the following sections.

## 1.2 Aim

The aim of the study is to build up a decision-making model that allows to assess the market value of the considered company with respect to the opportunities to expand investments and abandon (sell) capacities. Specifically, this paper is aimed to address two research questions:

1) How will the options to expand and to abandon affect the present value of the analyzed company if both of them are implemented separately and together?

2) How will the value of the analyzed company and the values of the embedded options change in response to changes in key variables, such as rate of corporate tax, discount rate and volatility?

## 1.3 Outline

The remainder of the paper is organized as follows. Section 2 provides introduction into theoretical perspectives underlying the structure of a model utilized in analysis. It is divided into four parts. Part 2.1 is intended to describe the properties of conventional NPV analysis as a precursor to the more sophisticated valuation models incorporating options. In part 2.2 the nature of options as financial derivatives is presented. The actual framework enabling to give a mathematical interpretation of the real options value is reviewed in part 2.3. Closing the theoretical section is part 2.4 where readers are proposed to get to know the basics of the time series analysis and statistical regression. The results of this part are essential to define the value of volatility, a major independent variable used in the analysis section.

Section 3 that focuses on the method of investigation is also portioned into several parts. Part 3.1 discusses the mainstream directions of social research and problems that accompany them. Concrete approaches, techniques and material equipment that assisted author in gathering and analysis of empirical data are developed in part 3.2. The limits of the study imposed both empirically and theoretically are presented in part 3.3. In part 3.4 it is explained what pivotal motives are behind the selection of the object of case study.

Section 4 reveals the details of the data exposed to numerical analysis. Again, it is more convenient to organize this section by separating it in two independent parts. Part 4.1 provides more comprehensive description of the company used as the object of case study. In part 4.2 the results of statistical regression and conventional NPV analysis are given.

Section 5 is provided completely to address the research questions raised to underline the aim of the current investigation. It starts with explanatory comments that clarify the content of some elements utilized in the model. The actual analysis and discussion are grounded on the facts and statistics that extend the findings of conventional NPV analysis. That is, they are aggregated in the form of extended NPV analysis. Ending the paper are suggestions for future research that can complement and enrich the material collected and processed in the course of the current work.

# 2. Theory development

## 2.1 The net present value analysis

Theoretically, ROA is a natural extension of the net present value (NPV) analysis that for many years determined the behaviour of investors. NPV represents a measure of profitability that accounts for the fact that the money expected to be earned in the future lose its value comparably to the current time (Brealey and Myers, 2003). Practically, there are two elements that should be disclosed to make the calculation of NPV possible, namely *free cash flow* (FCF) and *discount rate*.

### 2.1.1 Free cash flow

FCF is chosen to gauge financial performance of a business. It should be estimated based on the information from a standardized set of accounting statements. Following the collection of documents retrieved from the case company, FCF is measured by the formula:

$$FCF = EBITDA - interest\ payments - investments - corporate\ tax \qquad (2.1.1)$$

In the above expression, $EBITDA$ stands for *earnings-before-interest-taxes-depreciation-amortization*. That is, a share of revenue left after production costs and operating expenses are subtracted. *Interest payments* assume expenditures to extinguish borrowings. *Investments* show, in total, changes in capital cost spending and working capital. *Corporate tax* is a levy imposed in reliance with the regional tax regulations for businesses (Chartered Financial Analyst Institute, 2009). More detailed description of the structure of each element in respect to the object of analysis will be given in the empirical part of the paper.

### 2.1.2 Discount rate

Discount rate is a second important parameter of NPV analysis used to adjust the value of FCFs in respect to a particular time period. Financial literature fails to provide a versatile recommendation helping managers to find out an “exact” value of discount rate. Often, discount rate is equalled to the *risk-free rate*, which, by widely-internalized convention, takes the value of a coupon attached to a default-free governmental bond. Since even the obligations of developed countries are exposed to some degree of the risk to default, risk-free rate can be adjusted by the amount of premium for risk associated with doing business in a particular environment (Brealey and Myers, 2003). For this paper three values of discount rate will be used: risk-free rate, rate offered by the analyzed company and another rate selected arbitrarily.

### 2.1.3 Defining the present value of a business

Because of the obvious constraints to analytically predict future financial gains for the projects with the unspecified time of realization, a time period during which the fruits of the business activity could be reasonably observed is limited to 5-10 years. At the same time, any business entity of such a kind is deemed to be a going concern, i.e. assumed to be functioning within the unidentifiably long time period (Alexander *et al*., 2004). For this reason, financial theory prepared a recipe to approximate FCFs supposed to be received beyond the reasonably observable time period, i.e. FCFs in *horizon period*. According to Brealey and Myers (2003), an expression to gauge the total value of business is:

$$PV_{business} = PV_{observed} + PV_{horizon} \quad (2.1.2)$$

Here, the present value of a business in the observed time span implies the sum of the expected FCFs discounted in discrete terms. The first year, when FCFs become subject to discounting, is usually year 0 or 1. The expression is as follows:

$$PV_{observed} = E\left(\sum_{t=0,1}^{n} FCF(t)(1+r)^{-t}\Delta t\right) \quad (2.1.3)$$

Here, $n$ – the number of periods in the observed time span, $t$ – a specific period of time, and $r$ – discount rate (risk-free or risk-adjusted rate). In order to define the present value of a business beyond the observed time span, the first-year expected FCF in horizon, which follows right after the last expected FCF in the observed time span, should be adjusted by the discount rate calculated as a difference between the risk-free (risk-adjusted) rate and the rate of growth ($r_g$). Then, the resulting value should be multiplied by the discount factor used for the observed time period:

$$PV_{horizon} = (1+r)^{-n} * \frac{FCF(n+1)}{(r-r_g)} \quad (2.1.4)$$

When the present value of a business is known, it is time to evaluate its NPV by comparing the present value to the value of investments. It is stipulated that the company or project should not be commenced if their NPV is negative, i.e. investment expenditures exceed future returns (Brealey and Myers, 2003).

Unlike conventional NPV analysis, a valuation process based on real options embeds into the model an ability to effectively respond to the unexpected changes in the expected FCF patterns that may profoundly impact the profitability of a business project, i.e. make it more appealing in terms of financial reward. Consequently, after the introduction of real options, NPV takes the following form:

$$Extended\ NPV = NPV_{no\ option} + Value\ of\ option(s) \quad (2.1.5)$$

The next section will focus on the basic concept of option as a financial instrument, draw a line between the major types of options and describe key variables impacting the options value. Eventually, it will be shown how the achievements of financial theory in respect to financial options could be extrapolated to the real-world economy.

## 2.2 The general nature of options

### 2.2.1 Options in finance

Historically, the term *option* refers to a financial derivative that gives the right, but not the obligation to buy or sell some amount of a tradable security (usually, the stock of a listed company) at a pre-determined price at some timepoint in future (Hull, 2008). From this definition springs the basic separation of options in two categories. First one encompasses the derivatives allowing to purchase a security which are called *call options.* Second category unites the derivatives granting the right to sell a security, which are called *put options* (*ibid*).

Depending on a time point when options are exercised, they are also categorized as *American* and *European* ones. The former, according to the terms of the contract, could be exercised well before the latest possible date so long as the market conditions are favourable enough to justify this action. The latter, conversely, are devoid of the possibility of the early exercise and are therefore tied to a concrete date (Copeland and Antikarov, 2003).

As a result, a valid option contract should contain exhaustive information on such terms, as the current price of an underlying security that serves as the exercise (strike) price, the expiration date beyond which the contract loses its validity, and the exercise specifications that determine a degree of freedom granted to investors in respect to the earlier realization (Hull, 2008). Taken together, these terms determine the maximization strategy that investors should follow to benefit from signing the derivative contract. For the call option, the gain is maximized if the future price of a tradable security exceeds its exercise price. Accordingly, the call option is left unexercised if the strike price at the expiration date is higher than the actual security price. In this scenario, the only loss an investor undergoes boils down to the price of the option contract paid initially to a seller of the option. Formally, an expression for this strategy with the expected security price denoted as $S_t$ and the exercise price denoted as *K* is given as follows (Hull, 2008):

$$Max\ (S_t - K, 0) \tag{2.2.1}$$

For the put option, the maximization strategy is a reverse to that for the call option. The put option becomes worth exercising if the expected price of an underlying security falls below the level of the strike price. Under these circumstances, the holder of the option can sell her devalued security at the strike price to hedge against the factors that caused the devaluation. By analogy, the put option contract remains unexercised if an underlying asset is priced higher than its exercise price, and the investor loses some amount of money paid to a writer of the option agreement. By Hull (2008), the investor's utility is maximized like this:

$$Max\ (K - S_t, 0) \tag{2.2.2}$$

Using the lexicon of financial option theory, both call and put options that prove valuable to be exercised are referred to as being *in-the-money*. Otherwise, the derivatives are deemed to be *out-of-the-money*, and should be left unexercised. There is also a third outcome, when the strike price exactly equals the price of the underlying, which results in this option being called *at-the-money* (Copeland and Antikarov, 2003).

### 2.2.2 Extrapolating financial options to the real-world economy

Investors in their attempts to reduce the impact of uncertainties associated with projects in real economy had managed in adoption of financial options theory. The experience garnered allows to draw wide parallels between the framework that underlies the securities trading and the framework that defines real capital investments. The *strike price* in respect to the financial derivative is analogous to the investment expenditures incurred to start up any real-economy project. *The price of an underlying security* mimics expected cash flows to be generated in the course of the project realization. *The expiration time* of the financial option is equalled to the time allotted for the development of a real-economy project. Intuitively, real economy consists of those projects, which realization time could be credibly forecasted and those without the predetermined time boundaries. These are the European and American real options respectively (Copeland and Antikarov, 2003).

Similar to financial theory, taxonomy of real options includes a vast amount of models that reflect the nature and the objectives of considered investments. Among the call-type options, the frequently-used models present the rights to *delay* initially planned investments and to *expand* currently utilizing capacities. On the put side, there are opportunities to *conserve (scale back)* the part of undertaken investments or to *completely abandon* a project. In reality, these opportunities rarely appear alone and should be taken into account in combinations. For example, establishing a business gives an entrepreneur in future the opportunity to both expand and to abandon depending on the market situation. This sequence of options is termed *compound options* (Copeland and Antikarov, 2003).

For this paper, attention will be predominantly given to the options to expand and to abandon as those fitting in the investment program, whose details are given in the empirical part. The examples of general application of the chosen options will be also presented later, after the introduction of a theoretical model.

### 2.2.3 The value of a managerial flexibility

The value of a real option is a function of the value of an underlying asset that can be affected by several variables. First of all, there is a huge impact caused by the *value of an* *underlying* itself, which in this paper is the present value of expected cash flows. According to Copeland *et al*. (2000), the value of an embedded opportunity positively correlates with the asset value implying that the higher present value of a project leads to more valuable option. Second, the option as a part of a real-economy project is also highly contingent on *time to expiration*. Temporally protracted investments implemented in a phased manner often take years and increase the value of the timing options (such as option to delay or to expand). It relates to the fact that decision-makers are able to adequately change the configuration of the project in response to the market signals. A third factor that can drastically influence the option value is *volatility*. Volatility applies to changeability in cash flows that vary in respect to price and demand fluctuations. In a highly-volatile world investment projects can end up earning higher rewards that also spurs the option value up (*ibid.*). Later in this paper, a statistical approach employed to measure volatility based on time series will be discussed. At the same time, there are also variables, such as *capital expenditures* and *dividends* (or cash flows lost due to the project delay), that should be kept reduced to maintain the value of the option (*ibid.*).

As pointed out earlier, a single option project is rather an abstraction. In the globalised economy most investments are realized as bunches of different opportunities that together constitute what Trigeorgis (1993) describes as a *managerial flexibility*. Embedded in a combination, these opportunities are still exposed to the factors that primarily determine their values. But in addition, they become susceptible to so-called *interaction effects* that reflect the outcome of the merger and could be seen as significant contributors to the aggregated value. There is no clear-cut theory that enables to thoroughly explain the nature of interactions between the multiple options. Instead, decision-makers are proposed to be aware of some trivial dependencies that emerge between the options due to their combinations. Generally, uniting several options leads to a greater overall value of the project. But, as a by-product of the flexibility, this overall value is non-additive, i.e. it is below the aggregated value of the same options implemented separately. According to Trigeorgis (1993), there are following factors impacting the options interaction process:

*1) types of options embedded and their order of sequence.* Two call options integrated in a sequence (e.g. the option to delay compounded with the option to expand) lead to a more beneficial effect for a business, known as *super-additivity*. Accordingly, the effect of non-additivity emerges more profoundly if a call is written on a put and vice versa.

*2) the degree of overlap between the exercise times.* Two European options whose maturity periods coincide are expected to grow more valuable than the combination of opportunities with significant time gaps between the expirations.

*3) the relative degree of being in- or out-of-the-money.* This factor is in part related to the previous one and reflects the possibility of simultaneous expiration of several options leading to greater benefits.

## 2.3. Numerical methods of the real options valuation

This section introduces a theoretical model to compute the value of a managerial flexibility applying information available in financial statements of the analysed company. The focus is placed on the *Binomial option pricing model* that so far has several interpretations hinging on the determinants used and assumptions made (Chance, 2007). For this paper, the version prepared by Jarrow and Rudd (1983) and supplemented by Jarrow and Turnbull (2000) is given a preference since the structural elements used in its composition allow to better reveal the whole potential that the empirical information contains. Comparably to the original source, this dissertation uses different notations for some components that by no means impact the content and validity of the model.

Before the model will be presented, an important digression should be made to justify the utilization of some basic theoretical arguments. In the theory of financial derivatives, options are valued in respect to the price of an underlying security, which is freely tradable in the market. ROA in turn grounds its findings upon the present value of expected cash flows. That, actually, poses a challenge allowing for the fact that the cash flows are subject to the impact of multiple contributors, including prices for inputs and outputs, demand for output, fluctuations of interest rates etc. Despite some bold attempts to cope with this cumbersome complication (see Copeland and Antikarov, 2003; Godinho, 2006; Lewis *et al*., 2008), academicians generally concur that revenues used in the real options valuation follow the same pattern of behaviour demonstrated by the stock prices. Hence, mathematical concepts adapted to characterize the stock price movements are also applicable to cash flows.

### 2.3.1 Lognormal distribution of returns on stock prices

It is a widely accepted argument that the evolution of stock prices can be easily emulated using the model of the lognormal distribution. This model assumes that the logarithmic returns or percentage changes in stock prices are normally distributed in time (Jarrow and Rudd, 1983). In heuristic terms, the model has the following view:

$$Log\left(\frac{S_{t+\Delta t}}{S_t}\right) = \mu\Delta t + \sigma\sqrt{\Delta t}Z \tag{2.3.1}$$

For the sake of further analysis, the determinants used in the above expression should be shortly described. $S_t$ and $S_{t+\Delta t}$ are the current price and the price one period onward respectively. Other notations highlight next information (Jarrow and Rudd, 1983):

*$\mu$ - the mean logarithmic stock return per time unit measured as* $\alpha - \frac{\sigma^2}{2}$

*$\alpha$ – the geometric mean stock return per time unit (drift)*

*$\sigma$ – the standard deviation (volatility) of the logarithmic stock return per time unit*

*Z – the standard normal random variable, known also as the increment of a Wiener process, with mean 0 and standard deviation 1*

The last variable is a stochastic (random) one that can be encountered in literature under the title *white noise*. All the variables belonging to the white noise class are modelled as identically and independently distributed. Main properties that characterize their nature include zero mean ($\mu = 0$) and constant variance ($\sigma^2$) (Gujarati, 2004).

The presented lognormal distribution model that assumes the existence of discrete time intervals can be generalized in the form of *the geometric Brownian motion with drift* derived using the continuous time technique and stochastic calculus. It is given as follows:

$$dS = \alpha S dt + \sigma S dz \tag{2.3.2}$$

Here it is shown how the price of stock (*S)* evolves continuously in respect to the drift ($\alpha$) and the volatility ($\sigma$) (Dixit and Pindyck, 1994). Considering the huge potential that this model renders in the valuation of real options it might be of interest to investigate it in future, but for the current paper no additional attention will be given to it.

It should be noticed that the lognormal distribution is a state of nature that the stock price is expected to reach in the limit, i.e. when the time intervals between the two consecutive changes in price are so negligible or close to zero that the trading process is deemed to be uninterrupted (Jarrow and Rudd, 1983). The binomial option pricing model (BOPM), as a more simplistic tool, from the beginning does acknowledge the discrete nature of the price motion. Still, at the end of the day, it implies nothing but an approximation of the lognormally distributed process. Assumptions made by Jarrow and Rudd (1983) enable to prove this argument thoroughly.

### 2.3.2 The binomial option pricing model

In its original form the BOPM is intended for the securities valuation in a frictionless market with risk-free rate ($r$) constant over the option life period and the underlying paying no dividends. Investors are also presumed to concur to the fact that the stock prices tend to follow *a multiplicative binomial process* as given below:

$$S_{t+1} = \begin{cases} exp(u)\, S_t & with\, probability\ (p) \\ exp(d)\, S_t & with\, probability\ (1-p) \end{cases} \tag{2.3.3}$$

It reads that a period from now the stock price either increases to $exp(u)\, S_t$ with probability ($p$) or declines to $exp(d) S_t$ with probability ($1-p$). Here also $u > r > d$ and $1 > p > 0$ should be satisfied to eliminate the arbitrage opportunity, i.e. such a situation when the gain could be reaped as a consequence of a simple buy-and-sell operation using the disparity in prices in different markets (Jarrow and Rudd, 1983). The given process is also called *two-state* and underlies the structure of the *binomial lattice (tree)* – a graphical model that reproduces the two-state movements for later periods to let valuation of real options with the unspecified maturity time (Cox *et al*., 1979; Koller *et al*., 2010). Since the asset price develops constantly over time, two periods from now both states that the price has taken before are again subject to the probabilities of the similar upside of downside walk (Figure 1).

Further progress into future enables to formalize the stock price evolution as follows:

$$S_T = S_t\, exp(ju + (I - j)d) \;\; with\, probability \;\; \binom{I}{j} p^j (1-p)^{I-j} \tag{2.3.4}$$

**Fig 1. A 2-year binomial tree showing the evolution of the stock price**

| year 0 | year 1 | year 2 |
|---|---|---|
| | | 4) $exp(2u)\, S_{t,}\ (p^2)$ |
| | 2) $exp(u)\, S_t, (p)$ | |
| 1) $S_t$ | | 5) $exp(u+d)\, S_t, p(1-p)$ |
| | 3) $exp(d)\, S_t, (1-p)$ | |
| | | 6) $exp(2d)\, S_t, (1-p)^2$ |

Here, on the left-hand side, $S_T$ accounts for the stock price at maturity and *j* shows the number of upside steps that happen over *I* periods up to expiration. The right-hand side is termed *a complimentary binomial distribution function* for upward jumps (Cox *et al.*, 1979; Jarrow and Rudd, 1983).

When the stock price in the final year of a binomial tree is defined, the approach based on a backward induction should be used to find out the option value at each branch preceding the expiration period (Brealey and Myers, 2003). As mentioned earlier, the maximization strategy regarding a call option at its maturity is simply a difference between the stock price and the strike price, and the option remains unexercised if the latter exceeds the former (Hull, 2008). However, a time unit back, when the uncertainty conditioned by the left period prevails, the value of the call should be estimated in respect to the payoffs ($S_t$ – *K, 0*) earned at expiration.

So far, within the BOPM exist two approaches that measure the option value in a backward induction manner. Within the *replication portfolio* method the value of an option should be exactly equalling the value of an artificially constructed package comprising of some amount of stock and bonds. The parity between the values of both option and portfolio is a requisite condition enabling to avoid the arbitrage opportunity, which is prohibited in the state of equilibrium (Hull, 2008). Additionally, decision-makers are proposed to make use of the second approach, which is based on the *risk-neutrality argument*. Similar to the no-arbitrage idea that benefits from rather a desired state of economy, the risk-neutral world is a highly virtual reality with investors taking on an indifferent attitude toward the risk associated with participating in uncertain projects. In terms of financial analysis, it means that the risk-related premium assumed to be attached to risk-free rate is simply ignored. As a consequence, options in the time units preceding maturity appear to be the expected values of the terminal payoffs discounted at risk-free rate (Jarrow and Rudd, 1983). In practice, both methods carry identical outcomes, but in the name of technical simplicity, the risk-neutrality approach is preferred for the current paper.

Then, returning to Jarrow-Rudd's binomial model, value of a call is given the following form:

$$C_t = exp(-rh)\, E[\max(S_t - K, 0)] \tag{2.3.5}$$

Here, $C_t$ is the value of a call, $exp\ (-rh)$ represents continuous discount factor with $h$ showing the time to maturity, and the argument in the square brackets is the expected payoff at expiration. The distribution of $S_T$ that shows the price of stock at maturity (see expression 2.3.4) yields the general *binomial option pricing formula*:

$$C_t = exp(-rh) \sum_{j=0}^{I} \binom{I}{j} p^j (1-p)^{I-j} \max\,(S_t\, exp(ju + (I-j)d - K, 0) \tag{2.3.6}$$

All the components of the formula are known, except $u$, $d$ and $p$ that merit a special attention considering their essential role for the binomial valuation technique. According to Jarrow and Rudd (1983), in the limit ($I \rightarrow \infty$) their values are to be such that the stock price patterns shown as distributed binomially eventually start following the lognormal process as in (2.3.1).

The lognormal effect is achieved if (2.3.4) is slightly rearranged after taking the natural logarithm:

$$\log\left(\frac{S_T}{S_t}\right) = Id + (u-d)j \tag{2.3.7}$$

The factor $j$ (the number of upward jumps) in this expression is binomially distributed with the mean $E(j) = Ip$ and the variance $Var(j) = Ip(1-p)$. Following this, Jarrow and Rudd (1983) resort to the *theorem of DeMoivre-Laplace* to prove that for infinitesimally small intervals between each price movements both a binomial distribution of $j$ and a standard normal distribution inherent in the component Z converge:

$$\frac{j - Ip}{\sqrt{Ip(1-p)}} \approx Z \tag{2.3.8}$$

$p$ is arbitrarily chosen to equal 0,5. But, this assumption only holds in the limit (Chance, 2007) . Inserted into (2.3.8), it yields in respect to $j$:

$$j \approx \left(\frac{\sqrt{I}}{2}\right) Z + \frac{I}{2} \tag{2.3.9}$$

Substitution into (2.3.7) yields:

$$\log\left(\frac{S_T}{S_t}\right) = Id + (u-d)\left(\left(\frac{\sqrt{I}}{2}\right)Z + \frac{I}{2}\right) \tag{2.3.10}$$

To make the above result tantamount to (2.3.1), the determinants of the two formulas should be equated:

$$\mu\Delta t = \mu h = Id + (u-d)\frac{I}{2}$$

$$\sigma\sqrt{\Delta t} = \sigma\sqrt{h} = (u-d)\sqrt{\frac{I}{2}} \tag{2.3.11}$$

This is a pair of equations with two unknowns (*u* and *d*). It is solved to deliver the next formulas for measuring jump factors:

$$u = \frac{\mu h}{I} + \sigma\sqrt{\frac{h}{I}} \qquad \text{and} \qquad d = \frac{\mu h}{I} - \sigma\sqrt{\frac{h}{I}} \tag{2.3.12}$$

As a result, complete binomial process is as follows:

$$S_{t+1} = \begin{cases} exp\ (\frac{\mu h}{I} + \sigma\sqrt{\frac{h}{I}}) * S_t & with\, probability\ (p) \\ exp\left(\frac{\mu h}{I} - \sigma\sqrt{\frac{h}{I}}\right) * S_t & with\, probability\ (1-p) \end{cases} \tag{2.3.13}$$

Also, instead of the geometric mean stock return ($\alpha$) usually applied to reveal the mean logarithmic stock return ($\mu$), Jarrow and Rudd offer to use the risk-free rate ($r$) as a justification of the risk-neutrality argument underlying the call valuation, that is:

$$\mu = r - \frac{\sigma^2}{2} \tag{2.3.14}$$

It should be taken into account that these assumptions were initially worked out for the models with infinite time spans. In the situation, when the number of time periods is limited, these assumptions provoke arbitrage opportunities. As a result, the event tree built upon the reviewed BOPM does not recombine properly (Chance, 2007). Later, Jarrow and Turnbull (2000) prepared a redaction of the previous work to let the mathematical framework bring valid results also for the models within a restricted time perspective. It was revealed that the value of probability that converges to 0,5 in the limit deviates from this number when a limited time period is imposed. According to Jarrow and Turnbull (2000), the extent of the deviation depends on the volatility index ($\sigma$) and the length of a time period $\left(\frac{h}{I}\right)$. Therefore, the adjusted value of risk-neutral probability suitable to use in the BOPM can be estimated as follows:

$$p = \frac{exp\left(\frac{\sigma^2 \frac{h}{I}}{2}\right) - exp\left(-\sigma\sqrt{\frac{h}{I}}\right)}{exp\left(\sigma\sqrt{\frac{h}{I}}\right) - exp\left(-\sigma\sqrt{\frac{h}{I}}\right)} \tag{2.3.15}$$

### 2.3.3 Application of the binomial model in practice

Practical realization of the reviewed theory is selected to be shown in reliance with the example given by Koller *et al.* (2010). The starting assumptions for the example are given as follows. In the pre-flexibility scenario, a fictitious factory is assumed to generate cash flows with present value of $100 while investments are supposed to reach $105. Under the rules of conventional NPV analysis, the project should be rejected as future proceeds discounted at cost of capital (risk-adjusted rate) of 8% do not cover capital expenditures.

On the other hand, the project can be potentially improved in response to the market powers working both in favour of or against decision-makers. By the authors proposal, future opportunities taken to extend the original NPV value include options to expand and to abandon allowed to be exercised any time over five years. The expansion is planned to increase the project value by 20% and entail the outflow of cash amounting to $15 at each node of the binomial lattice. The decision to abandon is meant to bring the factory owners a lump-sum compensation totalling $100 at each node where the option to kill looks preferable comparably to carrying on. The opportunity cost due to the expansion delay (equivalent to dividends on a call option), as well as possible expenditures associated with the abandonment are omitted for simplicity. Both options are included in the model simultaneously, but also can be implemented separately.

In contrast to the version of the BOPM investigated for this paper, Koller *et al.* in the current example adopted the binomial framework proposed by Cox *et al.* (1979). In principle, both approaches are theoretically similar and computationally accurate enough to perfectly suit the two-state process that underlies the BOPM. However, minor discrepancies relating to the incorporation of variable components may lead to different results. Especially it applies to utilization of *drift* (α) completely ignored by Cox *et al.*, but embedded in the form of risk-free rate by Jarrow and Rudd. Practically, it means that the value of the underlying asset and accordingly the value of the assumed opportunities determined by Cox *et al.* appear to be somewhat underrated. It relates to the fact that drift (of course, if its trend is positive) naturally contributes to increasing value. Moreover, two approaches are also in discord in relation to the value of probability (*p*), which floats being dependant on risk-free rate and volatility in the model constructed by Cox *et al.* However, it is exclusively determined by volatility in the version of Jarrow and Rudd. For the current illustration, the results obtained reflect the theoretical findings of Cox *et al.*, but the forthcoming empirical part will be completely based on the Jarrow and Rudd's model.

The missing variables exploited in the example are volatility ($\sigma$) and risk-free rate ($r$) expected to have constant moments of 15% and 5% respectively in annual terms. By Koller *et al.* (2010), the present value of the assumed factory fluctuates either up or down as follows:

$$u = \exp\left(\sigma\sqrt{\frac{h}{I}}\right) = exp\ (0{,}15\sqrt{1}) = 1.1618$$

$$d = \frac{1}{u} = \frac{1}{1.1618} = 0.8607 \qquad (2.3.16)$$

As a consequence, the evolution of the present value over a 5-year period has a look as in Figure 2. When the lattice is completed, the values at each node are tested against the opportunity of expansion. That is, each value beginning from year 5 back should be multiplied by expansion factor 1.2 (since the expansion is expected to result in 20% increase) and the outcome then is reduced by the amount of $15 to account for capital costs necessary to perform the expansion. Then, after the expansion option is implemented, the lattice should be carefully scrutinized to pick up those nodes where both the no-flexibility value and the expansion-adjusted one are noticeably below the amount of compensation to be obtained because of shutting the factory down. In Figure 3 the nodes where going ahead with operations makes no economic sense are painted yellow. All the nodes below the yellow ones therefore automatically disappear as any development is aborted due to the earlier exercise of the abandonment option (Koller *et al.*, 2010).

The valuation of the factory with opportunities embedded is carried out in accordance with the risk-neutrality approach presented earlier. For instance, in the upper right node of year 5 the value chosen is the highest between the value to carry on amounting to $212 and the value after expansion equalling $239 (Figure 3).

| 0 | 1 | 2 | 3 | 4 | 5 | Item | Data |
|---|---|---|---|---|---|---|---|
| | | | | | | **Present value** | 100.00 |
| | | | | | 212 | **Discount rate** | 8.00% |
| | | | | 182 | | **Risk-free rate** | 5.00% |
| | | | 157 | | 157 | **σ (volatility)** | 15% |
| | | 135 | | 135 | | **Upside jump** | 1.16183 |
| | 116 | | 116 | | 116 | **Downside jump** | 0.86071 |
| 100 | | 100 | | 100 | | **Probability (p)** | 0.62861 |
| | 86 | | 86 | | 86 | **1-p** | 0.37139 |
| | | 74 | | 74 | | **Cost of expansion** | 15 |
| | | | 64 | | 64 | **Expansion factor** | 1.2 |
| | | | | 55 | | **Discount factor** | 1.05000 |
| | | | | | 47 | **Salvage value** | 100 |

**Fig 2. Evolution of the value of a factory with no option (Koller *et al.*, 2010)**

A slightly different maximization strategy is exploited in year 4, when the option-adjusted values of the project are expected values of the payoffs at expiration, i.e. year 5. Though, strictly speaking, the expiration time in this model is stipulated by the time boundaries imposed to ensure that the example is simple enough. However, virtually, the factory is assumed to be working beyond the 5 year limit. So, the value of $110 (at year 4) is selected as the highest between $100 that is simply the value without flexibility, $105 that is the value after the option to expand is introduced, and $110 that reflects the value of a wait for another year calculated as follows:

$$The\ value\ of\ waiting = \frac{((p*124)+(1-p)*100)}{1+r} \tag{2.3.17}$$

*(p)* in turn is found by the next formula:

$$p = \frac{(1+r)-d}{u-d} = \frac{1.05-0.861}{1.162-0.861} = 0.629 \tag{2.3.18}$$

It should be taken into account that the discount rate used here is discrete. Also, it turns out to be that in the model where no opportunity cost for the right to wait is presumed, it always makes sense to delay expansion up to maturity. This assumption is in complete accordance with the American call option that pays no dividends (Koller *et al.*, 2010). It can be seen that the value of the factory including options equals $114 which is $14 more than the present value derived by the conditional NPV approach. Therefore, with the options implemented, the factory yields the extended NPV which is high enough to cover capital expenditures at the outset, i.e. $105 .

| 0 | 1 | 2 | 3 | 4 | 5 | Item | Data |
|---|---|---|---|---|---|---|---|
| | | | | | | **Present value** | 113.53 |
| | | | | | 239 | **Discount rate** | 8.00% |
| | | | | 204 | | **Risk-free rate** | 5.00% |
| | | | 175 | | 173 | **σ(volatility)** | 15% |
| | | 150 | | 148 | | **Upside jump** | 1.16183 |
| | 129 | | 127 | | 124 | **Downside jump** | 0.86071 |
| 114 | | 112 | | 110 | | **Probability (p)** | 0.62861 |
| | 102 | | 101 | | 100 | **1-p** | 0.37139 |
| | | 100 | | 100 | | **Cost of expansion** | 15 |
| | | | 0 | | 0 | **Expansion factor** | 1.2 |
| | | | | 0 | | **Discount factor** | 1.05000 |
| | | | | | 0 | **Salvage value** | 100 |

**Fig 3. The value of a factory with options to expand and abandon (Koller *et al.*, 2010)**

## 2.4 Measuring volatility

The only parameter crucial in ROA that did not receive enough coverage in the previous sections is volatility. In terms of financial analysis, it is regarded as *"a statistical measure of the dispersion of returns for a given security or market index estimated by using the standard deviation or variance between returns from the same security or market index"* (www, Investopedia,1, 2012).

As discussed above, volatility in ROA should be applied to cash flows (revenues) that oscillate subject to numerous factors, such as prices for output products, input prices, changes in cost of capital etc. Academic literature is filled with recipes to measure the cash-flow based volatility using the Monte Carlo simulation (Haahtela, 2011). Despite their wide applicability, none of them is reliable enough as the dispersion of outcomes they generate, even in respect to the same case example, can be impressively huge (Godinho, 2006; Mun, 2002; Copeland and Antikarov, 2003).

Instead, the challenge of incorporating several sources of uncertainty can be reasonably simplified if assume that the cash flows follow the same pattern inherent in historical prices for the output produced. Then the approaches exploited to measure volatility in financial options analysis can be automatically applicable to valuing volatility in ROA. Given the extensive historical data containing the range of prices, volatility is simulated by the aid of econometric methods that deal with *time series*, i.e. such a type of data that includes random samples sequentially scattered over a certain time period (Gujarati, 2004).

Generally, any economic time series, observing evolution of indices like GDP (gross domestic product) or commodity prices in past, is represented by a set of stochastic variables collected at discrete time points. When taken in a set, these variables follow a certain path that can be categorized in terms of *stationarity*. According to Gujarati (2004), a time series is deemed to be *stationary* if its mean, variance and auto-covariance (at different lags) remain constant irrespective of a time point selected for assessment. On the contrary, *the non-stationary* time series has either mean, or variance or both changing over time.

Stationarity proves a crucial property of data series determining the framework that can be chosen to estimate different elements used in ROA. If the data set is stationary by its nature, it fluctuates by broad amplitude, but is always inclined to revert to some average value that requires to employ *the mean-reversion model* in further analysis. Vice versa, if the conditions of stationarity are not satisfied, the best way to represent the evolution of changes in the data set relates to the *model of random walk* (Gujarati, 2004). By preliminary observation, the random walk model also better suits empirical data gathered for this paper and it therefore should be given a closer look.

### 2.4.1 The random walk formalization

In its simplest form, the random walk process illustrates a short-term change in the variable ($Y_t$) subject to the only factor called *random shock* ($u_t$):

$$Y_t = Y_{t-1} + u_t \qquad (2.4.1)$$

Earlier in this paper, random shock has been interpreted in terms of the component of a Wiener process which is a category of stochastic processes normally distributed with zero mean ($\mu$ = *0*) and constant variance ($\sigma^2$).

As time goes by, the number of shocks accrues that flows into *the persistence of random shocks* phenomenon assuming that the influence of a particular shock that took place in past remains significant to the model (Gujarati, 2004):

$$Y_t = Y_0 + \sum u_t \qquad (2.4.2)$$

That is, the value of the variable *Y* at time *t* is equalled to its value at the beginning of the process added to the sum of random shocks at different periods. It is important to notice the mean and the variance that prove non-stationarity of the given random walk:

$$E(Y_t) = E(Y_0 + \sum u_t) = Y_0 \qquad and \qquad Var(Y_t) = t\sigma^2 \qquad (2.4.3)$$

Here the mean appears to be constant, but the variance increases indefinitely over time that cuts across the key assumption of stationarity implying time-invariant volatility (Gujarati, 2004).

Apart from the random shock, the value of the variable might be also exposed to *drift* ($\delta$) that extends the initial model by one component accounting for a possible long-term trend that the time series is supposed to take. The random walk with drift is usually schematized as follows:

$$Y_t - Y_{t-1} = \Delta Y = \delta + u_t \qquad (2.4.4)$$

Again, both the mean and the variance characterizing the model are expected to increase over time that is a clear evidence of non-stationarity:

$$E(Y_t) = Y_0 + t\delta \qquad and \qquad var\ (Y_t) = t\sigma^2 \qquad (2.4.5)$$

A synonym term oft-used in academic literature to describe the phenomenon of random walk is the *unit root process*. To explain the reasoning behind this name, the trivial expression of the random walk without drift has to be slightly augmented to resemble the Markov first-order autoregressive model, that is:

$$Y_t = pY_{t-1} + u_t \qquad where \qquad -1 \le p \le 1 \qquad (2.4.6)$$

As the above expression shows, the coefficient $p$ can float in the interval [-1,1] that impacts the view and the properties of the resulting formula. So, if $p = 1$, it shrinks back to the random walk without drift, i.e. a non-stationary stochastic process. However, if the equation $p = 1$ does not hold anymore, i.e. the absolute value of the given coefficient becomes smaller than a unit, the whole time series turns stationary. In the modified form, the examined expression is applied in the popular *unit root test* worked out to prove or refute stationarity of time series, and thereby justify application of the random walk model as such that underlies the stock price movements. Next section will be dedicated to the methodology necessary to both arrange a simple unit root test and derive the value of volatility.

### 2.4.2 The unit root test and standard deviation

In section 2.3.1 it was taken for granted that the returns on stock prices can be modelled in the form of lognormal distribution that is just a variety of the random walk with drift as might be noticed by comparing its pure expression in (2.4.4) with that one in (2.3.1). However, before the random walk process can be selected as a working model of analysis, it should be proved that the time series chosen actually is of non-stationary nature.

The unit root test allows to analyse any time series in respect to stationarity using computational capacities of the Microsoft Excel. The foundation of the test traces back to the model (2.4.6) presented a few paragraphs earlier. However, the actual version of the test workable in practice has a modified view as a result of subtraction of $Y_{t-1}$ from both sides of (2.4.6) (Gujarati, 2004):

$$Y_t - Y_{t-1} = pY_{t-1} - Y_{t-1} + u_t = (p-1)Y_{t-1} + u_t \qquad (2.4.7)$$

Equivalently, if $\varphi = (p - 1)$ the formula takes even more compact form:

$$\Delta Y_t = \varphi Y_{t-1} + u_t \qquad (2.4.8)$$

Thus, $\varphi$ is an actual coefficient of interest that should be equalled to zero, an assumption known as the *null hypothesis*. That is, the scenario when $\varphi = 0$ and $p = 1$ meaning the presence of a unit root serves as an indication of the non-stationarity of the time series. The time series is proved stationary if $p < 1$ and the coefficient $\varphi$ is negative (Gujarati, 2004).

**Table 3. Critical values of τ-statistics used in the Dickey-Fuller test (Gujarati, 2004)**

| Sample size | No constant | | Constant/No trend | | Constant and trend | |
|---|---|---|---|---|---|---|
| | 1% | 5% | 1% | 5% | 1% | 5% |
| **25** | -2.66 | -1.55 | -3.75 | -3 | -4.38 | -3.6 |
| **50** | -2.62 | -1.95 | -3.58 | -2.93 | -4.15 | -3.5 |
| **100** | -2.6 | -1.95 | -3.51 | -2.89 | -4.04 | -3.45 |

To figure out if the null hypothesis can be rejected, the coefficient $\varphi$ is checked against the critical values of the $\tau(tau)\ statistic$, a component of the multiple regression analysis. Those critical values (Table 3) are prepared in respect to the different regressions reflecting varied functional forms of the random walk process (Gujarati, 2004). By MacKinnon (1990), the three types of regression can be run, depending on the structural elements included. They are as follows:

$$\Delta Y_t = \varphi Y_{t-1} + u_t \tag{2.4.9}$$

$$\Delta Y_t = b_1 + \varphi Y_{t-1} + u_t \tag{2.4.10}$$

$$\Delta Y_t = b_1 + b_2 t + \ \varphi Y_{t-1} + u_t \tag{2.4.11}$$

The first one is based on a simple random walk model (no constant). For the second regression the previous expression is supplemented by the drift term (constant/no trend). Ultimately, in its fullest random walk has drift around a stochastic trend (constant and trend). The three are also referred to as the regularly versions of the Dickey-Fuller test (MacKinnon, 1990).

There is also an extended variant of the regression known in literature as the augmented Dickey-Fuller (ADF) test. Comparably to its regularly precursors, the ADF test recognizes serial correlation between the noise factors ($u_t$). Technically, it implies adding the lagged values of $\Delta Y_t$ to the above three regressions that can be formalized as follows:

$$\Delta Y_t = b_1 + b_2 t + \varphi Y_{t-1} + \sum_{i=1}^{m} a_i \Delta Y_{t-1} + \varepsilon_t \tag{2.4.12}$$

Here, $\varepsilon_t$ is the pure noise term and $\Delta Y_{t-1}$ illustrates shifts in the lagged variable over a time unit (Gujarati, 2004).

For the current paper, the regular Dickey-Fuller test accounting for drift as in (2.4.10) is performed using the Microsoft Excel computational framework. For the time series, consisting of stock prices ($Y_t$) and changes in logarithmic returns on those prices ($\Delta lnY_t$), the $\Delta lnY_t$ are to be regressed on lagged values of $Y_t$ applying the function "Regression". Cells containing $\Delta lnY_t$ should be selected for the field "Y range" and cells with the values of $Y_t$ are to be selected for the field "X range" (Orlov, 1996).

Volatility goes into the BOPM as a standard deviation of logarithmic returns on the output prices estimated through the function STDEV embedded in the Microsoft Excel. The function itself is based on the ordinary formula for estimation of standard deviation in statistics:

$$\sigma = \sqrt{\frac{\sum (Y_t - \bar{Y})^2}{(n-1)}} \tag{2.4.13}$$

Here, $Y_t$ relates to the return on output prices at time *t,* $\bar{Y}$ demonstrates the average value of returns from a certain time series and *n* is the time series size (www, Microsoft Office, 2012).

# 3. Method

## 3.1 Research in social science and the problem of trustworthiness

Historically, there are two mainstream approaches that determine both strategy and tactics of research in social science. The first one, ***positivism***, emphasizes the presence of objective world that proves independent of human conscience. *Facts* are a main source of knowledge that can be garnered from observations and experience. Based on the factual information, researchers are encouraged to discover laws that explain relationships between events and processes occurring in society. The links detected between two or more consecutive events are also called *constant conjunctions* and they constitute the foundation of scientific theory. In addition, facts gathered are given in a quantitative form and should be *value-free*, i.e. they should be exempt from individual preferences and believes of a researcher (Robson, 2002).

The second approach, ***relativism***, proves to be an alternative to social research. In contrast to positivistic view, it rather perceives surrounding reality as a projection of human mind that virtually makes impossible creation of the universal picture of the world. The role of facts, central in positivism, here is significantly downgraded as any researcher produces her own observations valuable enough to be regarded as "working hypotheses". Relativists accentuate the importance of *context,* within which a particular occurrence happens. Such elements of the context as language of study, personal values of individuals, morals and customs of society heavily matter when it comes to the understanding of the nature of events and processes. As a result, qualitative approach to data analysis is preferred (Robson, 2002).

The belonging to either scientific camp determines the design of research strategy. Proponents of positivism usually apply models of *fixed design* that are based on strong theoretical frameworks involving analysis of numerical data. It flows out from the name that researchers have a little space to manipulate the structure of a chosen framework. Instead, experiments that assume simulation of different components are enabled to confirm the viability and accuracy of the model. In this respect, researchers deal with *variables* that reflect properties of the object of investigation and which are subject to change (Robson, 2002). Depending on the role assigned to a variable, it can be either *dependent* that accounts for the result of observation, or *independent* that is exposed to manipulation to influence the value of dependent variable (Ary *et al.*, 2010).

Models of *flexible design* are traditionally a territory of researchers conducting qualitative studies. From the beginning, it does not require the existence of a well-established theoretical framework and therefore is perfectly tailored to those studies that target spheres of little knowledge. Comparably to fixed design strategies aimed at validating the workability of quantitative models, researchers utilizing flexible designs tend to rather give a comprehensive description of events and processes. In light of this, data collectors are not restricted in terms of the diversity of data that can be generated for the purpose of study. Although, it should be stressed that the information giving the insight of context is granted preference while numerical findings are proved supplementary. Popular categories of flexible design include case studies, ethnographic studies and grounded theory studies (Robson, 2002).

In reality, differences between the two concepts imposed by research theory are blurred. As Robson (2002) points out, majority of current studies prove to be a synthesis of both, i.e. they unite elements inherent in the fixed, as well as flexible frameworks. Such a type of hybrid research can be also applied to this paper. On the one hand, the analysis is based on the purely quantitative model that allows to estimate the value of a business with a managerial flexibility included. The value of an underlying together with the value of options here represent the observed or dependent variables. Whereas other reviewed variables of the model, including discount rate, cash flows, rate of corporate tax and volatility are independent ones that can be subject to simulation. Yet, the research is initiated to confirm the properties of the applied model that have been investigated and tested before in diverse environments. In other words, it is undertaken to prove that the value of both business and embedded options increases following the rise in volatility and reduction in discount rate, and decreases should the opposite occur.

On the other hand, this dissertation also contains the indications of qualitative approach. For instance, for the readers convenience some numerical information is conveyed graphically or structured in tables. In the literature such a way of data reporting is described as clearly qualitative (Robson, 2002). In addition, the preliminary source of empirical information is a real company that enables to call this paper a case study. According to Robson (2002, page 178), who used works of Yin (1981;1994) to synthesize the definition of case study, case study is "*a strategy for doing research which involves an empirical investigation of a particular contemporary phenomenon within its real life context using multiple sources of evidence*". Robson (2002) adds that the investigation in a certain case may benefit from the data that has both quantitative and qualitative roots. This fact allows to ascribe case study to the category of flexible or experimental design. Examples of "cases" that can be scrutinized include individual persons, communities, social groups, institutions, business organizations, countries etc. (Robson, 2002).

Irrespective of the approaches to social enquiry, there is always the challenge of veracity of data collected and results obtained. In primers on research methods this is often referred to as the problem of internal and external validity (generalizability) (Campbell and Stanley, 1963).

In short, the *internal validity* can be expressed as an evidence of a clear relationship that occurs between the manipulations performed and outcomes received. In the context of a fixed-design framework, it means the researcher's ability to prove that the changes in observed variables result exactly from simulation of independent variables. Cook and Campbell (1979) mention several factors that may undermine the internal validity of a qualitative study, but a few of them can be also related to any numerical model. For instance, *testing* is attributed to the threat of inconsistency between experiments conducted with time interruptions. It implies that the structure of the model conceived unchangeable prior to start of the series of tests can be arbitrarily altered by including or excluding variables. Another example is *ambiguity about causal direction*. This one questions the very presupposition that causes a researcher to call variables included in a model either dependent or independent. Internal validity is also exposed to such threats as *history*, *instrumentation*, *regression* etc. (Cook and Campbell, 1979).

The *external validity* indicates the degree to which the results of a particular study can be extrapolated to different theoretical or empirical circumstances. It is a well-known notion that the investigations based on case studies are hardly generalizable. However, it is mostly related to qualitative case studies that are highly dependent on the contextual details, while generalizability of case studies of quantitative nature remains somewhat under-examined (Merriam, 1985). At the same time, Robson (2002) argues that the results of a certain experiment can be externally valid within the social boundaries where the rules of statistical inference are applicable. Two additional strategies aiming to improve generalizability are relevant. The first one is *direct demonstration* implying the results of an experiment are empirically extended to other settings. The second way is *making a case*. It assumes that the characteristics of a given case study (object of research, time, setting and so forth) are compared to the same characteristics of other cases to prove similarity (Robson, 2002).

## 3.2 The gathering and analysis of empirical data

Research theory describes multiple methods of data collection that may benefit any study of either quantitative or qualitative type. At the same time, it is an internalized understanding that a qualitative investigation, set to rather interpret empirical findings (often collected in an unstructured form) than to confirm a theoretical model, requires more attention to the amount of collected information. The goal of a quantitative study is different since the raw data used should be of a pre-structured and processed type to suit the parameters of an analysed model (Robson, 2002).

This paper is mostly based on the information taken directly from financial statements of the company under consideration. A typical financial report in Ukraine consists of several enclosures, whose number vary depending on the size and the direction of a business. Specifics of steel industry are such that all the companies involved should prepare an extensive range of documents to stay in reliance with legislation. However, there are just three main forms with contents allowing to reflect a purely financial side of a company.

They include *balance sheet*, *income statement* and *cash flow statement* structured in line with the norms of international accounting practice (International Accounting Standards Board, 1997).

Clearly, the financial statements provided to allow for general interests of different stakeholders, should be additionally processed to meet the requirements of conventional NPV analysis. However, in preparation of data for the current paper, this stage was omitted as the numerical figures that do matter in calculation of both the NPV and the value of options were delivered to author in the form of a table drawn electronically in an Excel spreadsheet. The time series were also grouped and incorporated in the spreadsheet. The whole documentation set was obtained by email from a representative of the company. Complementary information to make clear some contextual details was collected during the interview with chief shareholder. The interview was of a *semi-structured* type as suggested by Robson (2002). For this type of interview questions were prepared and sent to the respondent in advance, but then were freely modified during the conversation. The interview was organized between the author, the respondent and his secretary and lasted for a half an hour. The result is digitally recorded.

The analysis of the gathered numerical data was carried out completely by means of the Microsoft Excel. The time series includes three sets of real (deflated) monthly prices for outputs; each of them covers a period of four years. They were collected to determine whether the prices for products of the given company follow a random walk or move stationary. For this purpose, all the three sets of prices were analysed using the function "Regression" pre-installed to exhibit the statistical properties of a multiple regression model (with one dependent variable and many independent ones). The ultimate report of the regression procedure represents a table divided in three parts: regression statistics, ANOVA and regression coefficients (Cameron, 2009). For the current analysis only the *t-statistics* coefficient is of interest. Still, the full results of the test and the time series analysis are available in Appendices 1 and 2.

The model of a binomial lattice similar to that one suggested by Koller *et al.* (2010) was constructed manually in Excel spreadsheet. Different pre-installed formulas enabling to link the spreadsheet cells and thereby facilitate the replication of the lattice were utilized. The way how the lattice was arranged allows to simultaneously represent the evolution of the present value and incorporate opportunities to expand capacities and abandon the market by selling the whole complex. In sum, three diverse templates of the lattice were prepared: in the presence of expansion option only, in the presence of abandonment option only, and with two options implemented together. To account for different combinations of independent variables, each model of the lattice was run 27 times. Due to the physical features of the lattice (enormous size, interactive components), its full representation on a standard sheet is unfeasible. Instead, simple spreadsheet tables containing the values derived in the actual lattices appear to be a reasonable substitution (Appendices 4,5 and 6). Still, curious readers can obtain the actual Excel-based lattices upon request.

The combination of independent variables exemplified by the appendices is as follows: tax rate 25%, discount rate 13% and volatility 29,5%. Appendix 4 illustrates the values of the template with expansion option. Each node, apart from the nodes at year 0 and year 10, is filled with three figures: the present value of the company, the value of the company after expansion, and the value of waiting. Nodes where the value after expansion is negative or below the present value and the value of waiting signal that the initiative to expand capacities cannot be fulfilled. This situation is true throughout the entire lattice, besides some nodes at year 10, when the choice to expand is preferable. This is due to the fact that the evolution of the project's value is technically invisible beyond year 10, though, it is assumed that the production process is actually not ceased.

The template with abandonment option from Appendix 5 includes nodes with the present value and the value of waiting, but does not incorporate the value after expansion. Moreover, at some nodes both values are replaced with the only sum that shows the amount of funding that the company receives if the current capacities are sold out, i.e. the option to abandon is exercised. All the nodes positioned below the nodes with abandonment are considered to be inactive and labelled NE (not exist).

In Appendix 6 both options are incorporated simultaneously. Practically, this template reproduces the values firstly generated in the expansion procedure. Then, each node is tested against the opportunity to abandon. It is carried out by using the backward induction technique from the nodes at year 10 and to the beginning of the lattice. At year 10 the return from abandonment is compared to the present value and the value after expansion. Starting from year 9 back, the value of waiting is also taken into consideration. The maximization strategy is straightforward: if the three values are below the value of compensation offered due to abandonment, the operations are terminated. Consequently, the sequence of the options is such that the opportunity to expand precedes the opportunity to abandon. That is, the abandonment option is effectively written on the underlying, which is prior exposed to the expansion arrangements.

As stated in the aim, the purpose of this study is not only to measure the influence of embedded options on the company's value. Additionally, it is of interest to show the reaction of both the value of the underlying and the value of the options in response to shifts in rate of corporate tax, discount rate and volatility. In financial literature such a manipulation with independent variables undertaken to test the behaviour of dependent one is known as *sensitivity analysis*. More precise definition is given by Saltelli *et al.* (2008, page 1): *"the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input"*. In its simplest, sensitivity analysis demonstrates the variation of output when the only element of input is changed per time while others are kept constant (Brealey and Myers, 2003).

A variation of sensitivity analysis is *scenario analysis*. The difference between them lays in the fact that in sensitivity analysis the range of possible values of an independent variable subject to simulation is selected arbitrarily, while for scenario analysis the values of the variable are available in a market or calculated based on the primary data (www, Damodaran Online, 1, 2012). In the context of this paper, a few values are chosen on the basis of a subjective vision of the market future, but the selection of others is substantiated by the relevant sources. More specifically, the range of rates of corporate tax, except for one given in the company's report, is selected arbitrarily. Two rates of discount are strictly data-stipulated, and the third one is chosen as a counterweight to the first two. Finally, all the three values of volatility are estimated on the basis of time series representing prices for different outputs. In the next sections, it will be shown what values are actually assigned to different variables.

## 3.3 Delimitations

The study undertaken in this paper is delimited both theoretically and empirically. Considering the fact that the study is based on the data from one company, it entails some challenges in terms of generalizability of the results to other investment projects. At the same time, these challenges are exclusively data-specific, and are not attributed to the theoretical framework as a whole. It simply means, that numerical figures dug out from other businesses or industries, wherever they are located in the world, will certainly produce different outcomes, but the cause-effect links installed in the model remain unaffected. So, in this sense, the study may be regarded as generalizable.

Regarding a theoretical basis, this paper relies on the option pricing model organized in the form of two-state (binomial) process. Among the versions of the framework, represented in literature, it is decided to choose that one proposed in works of Jarrow and Rudd (1983) and Jarrow and Turnbull (2000). Reasons behind selecting this particular model refer to the mathematical capabilities that enable to reveal the whole potential enclosed in the empirical information. Other theoretical frameworks in respect to real options valuation are touched partially or completely ignored. Similar arguments apply to the selection of the methodology of volatility estimation. Among the alternatives available, the chosen one is the most widely-used that yields reliable results and it does not require sophisticated software to deal with.

This paper is also quite selective when it comes to the number of real options taken into account. The empirical information that author has managed to collect allows to concentrate on two options, including option to expand and option to abandon. Literature is abundant with other alternatives to consult, but not all of them fit into specifications of the investment project taken as a case. The one more managerial opportunity potentially suitable to consider, but discarded due to the lack of necessary data, is option to switch. It will be given a closer attention later in the section on the future aspects of research.

The way how the independent variables are treated in the course of analysis is another factor of delimitation. A few of the independent variables, such as rate of corporate tax, volatility rate and rate of discount are subject to simulation, i.e. they are changing in the intervals imposed based on the most probable scenarios of their behavior. At the same time, a group of independent variables consisting of the factor of cost-price squeeze, cost of investment in expansion and cost of abandonment are deemed to be fixed through the process of analysis. Volatility and discount rate are selected to simulate because they are key variables in achieving the aim of this investigation. Rate of corporate tax and the factor of cost-price squeeze are equal in terms of their contribution to analysis, but the first one is preferred since it is induced by government capable of correcting the market forces. Cost of investments in expansion and cost of abandonment are exempt from simulation for reasons of simplicity.

## 3.4 The choice of the object of case study

The study undertaken for this paper revolves around a single business entity, *Metals and Polymers Ltd*. This company is located in Eastern Ukraine and specializes in production of protected steel, namely galvanized steel and polymerized steel. There are several objective reasons to justify this choice.

First of all, the company is an interesting example from theoretical perspective. Shareholders have initially seen it as a multi-stage investment project that assumes the pilot stage to experience the influence of market environment in action, and the expansion stage that implies further development of installed capacities so long as the market conditions favour it (Pers.Com., Risukhin, 2012). Speaking the language of real options theory, by seeding capital to give the project a go investors have thus written a call option without a predetermined exercise moment (Hull, 2008). In other words, there is a possibility to evaluate the profitability of the given business using the toolkit of real options analysis.

Second feature crucial to this study is *uncertainty*. The project under consideration is exposed to different sources of uncertainty. On the one hand, there is a volatile market environment with prices for both inputs and outputs oscillating randomly. The presence of high volatility makes a big difference in ROA since the perspective of upside jumps in output prices adds value to both the underlying and the call option written on it. Whereas the threat of market going down is simply overlooked as an investor is not obliged to exercise the option under these conditions. At the same time, a high likelihood of slowdown makes it reasonable to include in estimation the opportunity to leave business at the price higher than the market one. That is, the option to abandon can be written.

Last but not least, the choice of the object of case study is also conditioned by the fact of personal contacts between author and the company's leadership. The importance of this factor is huge as the selected company is not a listed one and therefore it is not legally compelled to publish financial statements to independent stakeholders. Only those stakeholders having special authorities can demand an access to the company's data base.

# 4. Empirical background and results

This section is divided in two parts. First one is devoted to the object of case study, a steel mill. Details of the technological process of steel production are given. It is shown what ingredients are applied as inputs and what type of production they allow to obtain. Preliminary data on the intention to expand industrial capacities is discussed. In the second part, the results of the valuation of variables crucial in the following analysis are presented. In particular, the content of the spreadsheet with key financial data is thoroughly reviewed. The economic rationale behind choosing discount rates and rates of corporate tax is explained. The conventional NPV analysis is delivered as a scenario analysis with the value of the company tested against changes in taxation policy and different levels of market uncertainty. Ultimately, the outcomes of the time series analysis are summarized.

## 4.1 Description of the object of case study

*Metals and Polymers Ltd.* (further – Metals and Polymers) was established in 2008 in the town of Alchevsk, Eastern Ukraine (Figure 4). The company does not officially disclose the structure of shareholders, but it is known that its main share is owned by the firm *VR Holding ApS* registered in Denmark by Ukrainian beneficiaries (www, Danish Worldwide Trade Company, 2012).


**Fig 4. The map of Ukraine. The red arrow points to the location of the company** (www, World of Maps, 2012)

Currently the company operates industrial capacities capable of producing two types of output – galvanized rolled steel and polymerized rolled steel. Both of them belong to the big class of rolled steel, which is an intermediary steel product obtained in the process of rolling. To produce a rolled stock, a simple steel billet (slab) should be subject to pressing under temperature between a pair of moving rolls (Figure 5). Compared to other rolled products, these ones have a special anticorrosive cover to provide protection from different atmospheric effects and improve durability (Ilyin, 2009). Throughout this paper these products are shortly called galvanized steel and polymerized steel. A synonym «protected steel» is also used when the type specification is not essential. Technologically, the effect of protection is achieved by putting the anticorrosive layer on the surface of either cold-rolled or hot-rolled steel, which are the categories of rolled steel without rust prevention. Further in the text they are also labelled «unprotected» or «naked steel» to underline the contrast with protected products.

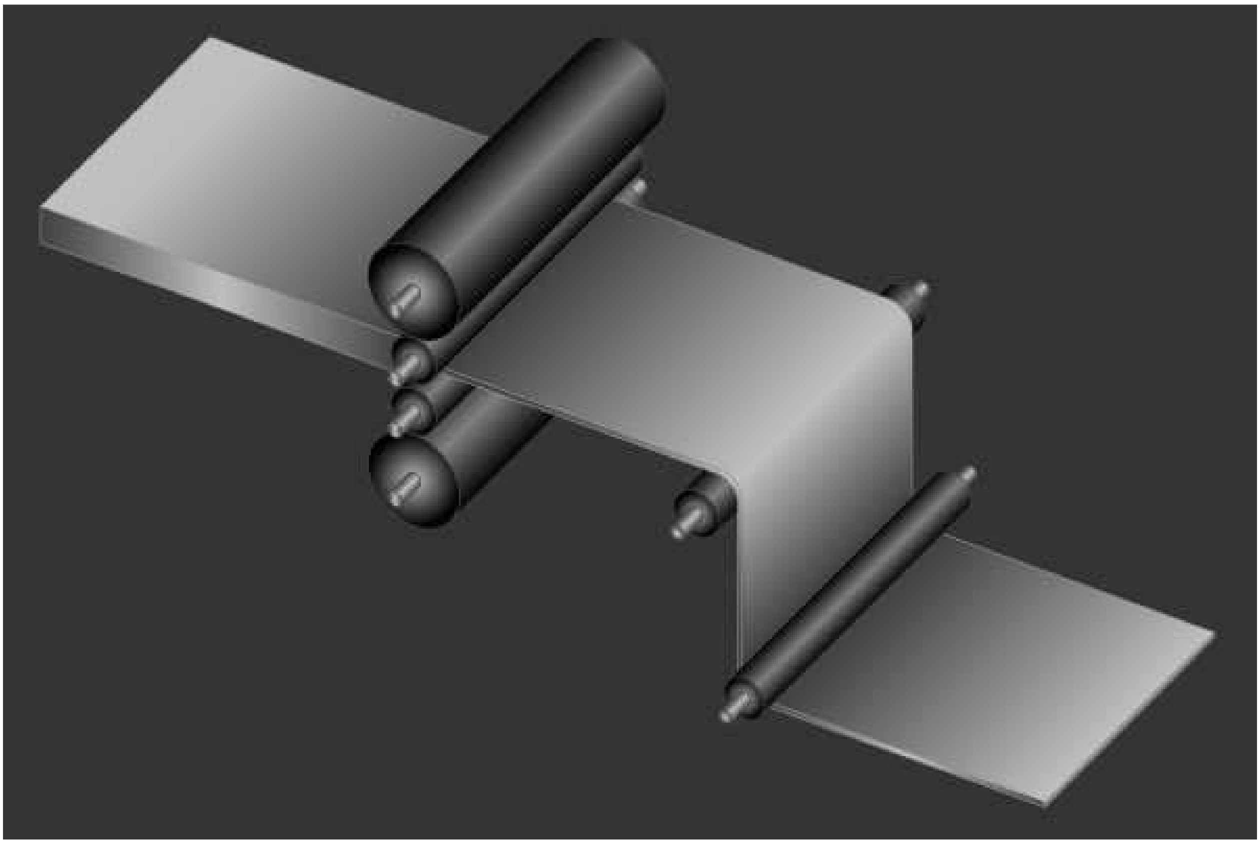

**Fig 5. The process of rolling a steel billet (slab)** (www, Arc Abrasives, 2012)

The whole technological process of providing protection is composed of two stages. First, unprotected steel should be coated with a layer of zinc and aluminium in a multi-phase process called *galvanization*. This stage itself provides reliable protection of naked steel and prolongs its service duration to 15 years. Second, the steel with galvanized protection can be also painted with a special polymeric solution to provide a decorative effect and make it even more resistant to the impact of atmosphere. The *polymerization* process enables to paint steel in diverse colours, which is an especially valuable characteristic in development sector. Again, both processes are technologically separated and should not be necessarily integrated into one complex. What is important is that galvanization always precedes polymerization. However, a company may easily specialize in production of either galvanized steel or polymerized steel. In the market, protected rolled steel is realized in coils packaged in polyethylene or paper and roped (www, Metals and Polymers, 1, 2012).

Thus far, the industrial complex of Metals and Polymers occupies the territory of about 17 hectares and includes three main departments: workshop of galvanization, workshop of polymerization (painting) and administrative building. The production lines are produced, supplied and assembled by Australian firm *Bronx International Pty. Ltd*. The aggregated capacities installed allow to annually produce 100 000 tons of galvanized steel and 75 000 tons of polymerized steel. Investments amounting to $72 mln were financed in a partnership between the company's founders and banking establishment. The borrowing provided by Alpha Bank (Ukraine) totals more than $40 mln. (www, Metals and Polymers, 2, 2012; www, Steel Orbis, 1, 2010).

To obtain output, several inputs should be mixed in production process. As a main ingredient, Metals and Polymers uses cold-rolled steel manufactured by Turkish companies *Tezcan* and *TATMetal*, and Russian firm *Severstal*. Annually, the company has to purchase 175 000 tons of cold-rolled steel to galvanize and colour, provided the production capacities are fully reserved. Zinc, aluminium and other chemicals utilized in both galvanization and polymerization are purveyed by *Henkel Surface Technologies* from Germany. From *Akzo Nobel Industrial Finishes AB* (Sweden) and *PPG Polifarb Cieszyn* (Poland) the company is contracted to purchase polymeric paint (www, Metals and Polymers, 3, 2012).

According to its leadership, Metals and Polymers was initially set up to met the demand for protected steel in domestic market where a few national producers compete with multinational suppliers from Russia, India, Finland and other countries. By the company's estimates, it is competitive enough to propose the production, which is cheaper than that exported from abroad, and which is of higher quality than the offerings from domestic rivals. As people in charge of the company explain, domestic steel producers are able to offer cheaper production at the expense of difference in wages, which in Ukraine are generally lower than in the countries-competitors. A superiority in quality in turn can be achieved due to the brand-new equipment installed and modern technologies adopted (Pers.Com., Risukhin, 2012). So far, in Ukraine function 150 firms focusing on the processing of polymerized steel and 250 firms that demand galvanized steel as an input. By consuming protected steel they produce building materials (sandwich panels, roof tiles) and components for electronic appliances. The parameters of production capacities enable Metals and Polymers to fulfil small-scale orders from households too, but long-term contractual collaboration with other businesses proved preferable (*ibid.*).

Taking into account stable growth in consumption of protected steel in Ukraine (Figure 1), the company considers the opportunity to expand current capacities in future. In the interview with author its chief shareholder said that they had purposefully chosen to construct the plant on a site, which has enough space for an additional complex of a comparable size (Pers.Com., Risukhin, 2012). It is assumed that the expansion stage would result in an increase in capacities for galvanization and polymerization. Moreover, the company hopes to develop by building up a pickling line and line to produce cold-rolled steel. The last two are needed to start manufacturing own cold-rolled stock and get a wider control over production cycle. This means that the dependence on outside suppliers of cold-rolled steel would be reduced or even completely eliminated, and the company would also get an opportunity to widen its production range. After expansion, the total production capacities would amount to 300 000 tons of rolled steel per year (www, Bronx International, 2012).

In financial terms, the expansion stage would cost Metals and Polymers $67 mln and would double its EBITDA. Based on the preliminary data available, the company managed to launch its production activities in the late 2010 and planned to start expansion in the period 2012-2013. Upon the inception of the current paper, financial results for year 2011 were not ready yet. Nor was it explicitly known whether the decision to expand this year would be discarded or approved. At the same time, the company's leadership sets no deadline for the expansion stage, making it clear that waiting is better than developing in wrong time (Pers.Com., Risukhin, 2012). Further details that shed a light on the company's perspectives to expand will be given in the following part.

## 4.2 Results

### 4.2.1 Examination of data from financial statements

All the data that allows to evaluate the profitability of the company from a perspective of conventional NPV analysis is collected in Appendix 7. It is a prognosis of financial results for a 10-year period made by the analysts of Metals and Polymers. This table reflects the vision of the market in pre-production time and does not contain the actual results obtained in years 2010-2011 when the production process just started. This omission is justified as in 2010 the production capacities mainly stayed idle and the forecast could not have been significantly impacted, while financial results for 2011 were not available yet before the work on this dissertation began. Still, only results of the pilot phase of investments are revealed, whereas the cash flows from expansion are not considered.

By construction, this table unites the elements of the income statement and cash flow statement. However, comparably to the classic versions of the documents recommended by accounting professionals, it is given in an abridged form with some components missing. For instance, there is no clue on the amount of production sold, revenue, cost of revenue and operating expenses that precede EBITDA and might have contributed to current analysis. In the absence of these variables, it is taken for granted that the production capacities are always fully loaded, and the only factor that may impact the amount of cash flows relates to the output prices.

It is shown in the upper lines of the table that the project starts generating cash inflows the same year when the pilot investments are made. However, in year 0 the level of EBITDA is just a half of that in forthcoming years, because it is presumed that the equipment installed is not loaded fully that period. Starting from year 1 the plant is expected to be working at full capacity, and EBITDA is assumed to double and remain unchangeable in future. As noticed earlier, this is a strong simplification implying that neither price fluctuations, nor changes in demand affect the amount of EBITDA. To make further analysis more realistic, it is supposed that EBITDA would be gradually eroding at the annual constant rate of 1.5%. This rate reflects the concern that the prices for the input components would be raising faster than the output prices leading to the squeeze of operating profits (the phenomenon of cost-price squeeze).

It is worth reiterating that the goal is to strip EBITDA of all the expenditures to find out the amount of free cash flows as shown by the formula (2.1.1). Down through the table, there is *interest* that shows the amount of debt emerged due to the capital borrowed from Alpha Bank (Ukraine). According to the plan, the debt should be amortized by unequal portions annually within the period of 6 years. Next element subject to subtraction from EBITDA is *investments*. It is expected that the company is guaranteed to have an access to the necessary financial resources to fully realize the first stage in year 0. Again, the full installation of capacities does not mean that they are completely reserved to fulfil orders. It can be seen that the capital costs are split in *kit and infrastructure* and *working capital*. The first group includes two production lines, factory building, land, transport vehicles, instruments, spare parts etc. The second one implies goods and materials needed to kick the production process off like electricity, water, fuel etc. EBITDA net of interest, investments and cost-price squeeze yields pre-tax cash flow.

### 4.2.2 The choice of tax rate, discount rate and their role in NPV analysis

The amount of a free cash flow utilized in NPV analysis directly depends on the rate of corporate tax that should be imposed on a pre-tax cash flow. The analysts of Metals and Polymers proposed the only scenario for its free cash flows based on the officially implemented rate of corporate tax that in Ukraine equals 25%. To highlight the role of the government as a powerful player capable to change the rules of the game in the domestic market, two additional rates of corporate tax for steel industry are introduced as possible alternatives to the existing one. On the one hand, the rate is increased to 30% if there is an initiative to boost the state budget at the expense of excessive profits that the steel producers can earn. The increase in the tax rate can be also interpreted as a possible spike in the amount of unofficial payments to the industry functionaries, known as bribes. On the other one, the rate can be decreased to 20% in case the industry encounters the downfall in the market and the state decides to step in and mitigate the implications. It should be added that these rates are arbitrarily chosen and they are not associated with any real-life initiative.

Besides tax rate, theory requires to determine the cost of capital to account for the risk of investments in a particular market environment. The discount rate provided by the company amounts to 9,5% which is in close reliance with an estimate promoted by independent analysts for steel industry globally (www, Damodaran Online, 2, 2012). At the same time, circumstances under which the current project is being realized demand more scrupulousness in selecting discount rate. Ukraine is considered to be a traditionally risky environment for doing business because of multiple uncertainties, such as the risk of losing property or difficulties in obtaining construction permits (www, Doing Business, 2012). Since the global estimate of the cost of capital for steel industry focuses rather on sectoral uncertainties common worldwide than on country-specific risks, the rate chosen for Ukraine can be seen too low from the viewpoint of an international investor. Simultaneously, this argument can be fairly criticized, for the considered company is founded and managed by Ukrainian professionals who are experienced enough to cope with the local uncertainties. To stay impartial, the range of possible scenarios in respect to discount rate is also widened to three ones. Apart from 9,5%, free cash flows will be discounted at 6,75% and 13% within the sensitivity analysis. The lower rate is a risk-free rate equalling the value of a coupon attached to the 10-year Eurobond issued by Ukrainian government in 2007, while the higher rate is selected arbitrarily by author (www, UFC-capital, 2011).

As a result, the complete conventional NPV analysis includes 9 different outcomes depending on the tax rate and the discount rate applied (Table 4). As expected, the highest profits are obtained in the presence of the lowest discount rates. Regardless of the taxation policy prevailing in the market, the project is worth undertaking as its NPV is well positive. The situation noticeably changes when the project's cash flows are discounted at rate 9,5%. The total present value under the varied rates of corporate tax is almost twice as less as in the case of low risk environment. Nevertheless, it is still high enough to cover the capital costs and consider the project profitable. The worst outcomes arise when discount rate increases to 13%. By the rules of NPV analysis, the project is too risky even if the tax burden is reduced to the minimum level assumed. In view of the fact that its NPV is negative, investors must refuse from the idea of undertaking the project.

It should be noticed that the growth rate used in calculation of the total present value is equalled to 3%. The origin of this number comes from the time series showing the evolution of monthly prices for different outputs produced by the company. The details of the time series analysis will be presented in the following section.

**Table 4. Distribution of present values for different tax scenarios (NPV analysis)**

Values in $ millions

| **Base case scenario: tax rate 25%** | | | |
|---|---|---|---|
| **Discount rate** | **6.75%** | **9.50%** | **13%** |
| **PV horizon** | 217,233 | 97,181 | 46,116 |
| **PV 10 years (observed)** | 24,537 | 12,079 | - 628 |
| **PV total** | 241,770 | 109,260 | 45,488 |
| **NPV** | 169,471 | 36,961 | - 26,811 |
| **Scenario lower tax: tax rate 20%** | | | |
| **Discount rate** | **6.75%** | **9.50%** | **13%** |
| **PV horizon** | 235,432 | 105,322 | 49,979 |
| **PV 10 years (observed)** | 34,434 | 20,892 | 7,067 |
| **PV total** | 269,866 | 126,214 | 57,046 |
| **NPV** | 197,567 | 53,915 | - 15,253 |
| **Scenario higher tax: tax rate 30%** | | | |
| **Discount rate** | **6.75%** | **9.50%** | **13%** |
| **PV horizon** | 199,034 | 89,040 | 42,253 |
| **PV 10 years (observed)** | 14,641 | 3,266 | - 8,322 |
| **PV total** | 213,675 | 92,306 | 33,930 |
| **NPV** | 141,376 | 20,007 | - 38,369 |

### 4.2.3 The stationarity test and estimation of volatility

It was earlier assumed that the future cash flows from the considered project follow the same path as the output prices. As a consequence, all the parameters derived to characterize the movements of prices can be automatically used to describe the evolution of cash flows. For this paper, three sets of prices for each type of production that Metals and Polymers manufactures or plans to manufacture after the expansion phase are gathered. In particular, these are the real (deflated) monthly prices for polymerized, galvanized and cold-rolled steel encompassing a 4-year time interval from January 2007 to December 2010. This data is retrieved from the company's internal archive and is denominated in American dollars, so is the other monetary information given in the paper. In addition, a set of monthly prices for cold-rolled steel covering a 17-year period (from January 1993 to December 2010) is collected from a public source to give a visual picture of volatility in the steel industry for the last decades (Appendix 3).

The graph in Appendix 3 proves that prices for all the types of production generally evolve in a similar manner. While the vertical difference is just an implication that the products are manufactured at different stages with polymerized steel having the highest added value. Here it also should be highlighted that the prices for cold-rolled steel from the longer time period are estimated in dollars per 100 pounds ($/cwt) that is a usual way of pricing commodities in some countries. The other three sets are prices in dollars per ton($/ton). It is evident from the graph that prior to the 2000s, the market was calm, and the steel prices fluctuated insignificantly showing a steady decreasing trend. Since 2002 volatility has been prevailing in the industry with prices raising to culminate in the summer of 2008 and then steeply fall a few months later.

The reason for collecting prices is to establish the data series applicable in the stationarity test as described in the theoretical part. If the data set is proved non-stationary, the random walk model is relevant in further investigation (Gudjarati, 2004). Besides, it is of necessity to find out both the volatility to plug in the model for option valuation and the growth rate required in derivation of the company's present value. Monthly prices for the given 4-year period constitute a series of 48 stochastic variables. The complete results of the multiple regression analysis are included in Appendix 2. Here, the summary of the important values are given separately in Table 5. Based on it, the non-stationarity of the three data series is quite clear as the values of τ-statistics are well above the thresholds suggested by Gujarati (2004) in Table 3. It means that applying the random walk model is justified.

Regarding the values of volatility and drift, the minor adjustments to formulas should be done. So, formula (2.4.13) is used to measure average monthly standard deviation while for this paper annualized volatility is demanded. To meet the requirements, the existing expression is multiplied by the radical of 12, i.e. the number of months per year (www, Investopedia, 2, 2012):

$$\sigma = \sqrt{\frac{\sum(Y_t - \bar{Y})^2}{(n-1)}} * \sqrt{12} \qquad (2.4.14)$$

The average (mean) of logarithmic returns should be also annualized before inserting it in the formula for drift. It is performed as follows:

$$\mu = \frac{\sum_{t=1}^{n} \Delta ln Y_t}{n} * 12 \qquad (2.4.15)$$

From the past sections it is known that $\Delta ln Y_t$ shows changes in logarithmic returns on prices and $n$ is the time series size. The value of drift is found using information from the expression (2.3.1).

**Table 5. Summary of the time series analysis**

| Type of output | Polymerized steel | Galvanized steel | Cold-rolled steel |
|---|---|---|---|
| Average monthly st. deviation | 6.66% | 8.52% | 10.63% |
| Annualized st. deviation | 23.06% | 29.50% | 36.83% |
| Annualized mean | 0.34% | -1.94% | 4.09% |
| Drift | 3.00% | 2.41% | 10.87% |
| τ - statistics | -1.41545 | -1.57589 | -1.33977 |

Table 5 clearly demonstrates that cold-rolled steel is the most volatile product in the given range. Currently, the company does not have it in its production portfolio, but its manufacturing is planned as a part of expansion strategy. Moreover, it is a main component of input that actually accounts for the risk of cost-price squeeze. Two other products are assumed to equally contribute to the volatility of cash flows, but in reality their weights may be different subject to fluctuations in demand, technical breakdowns etc. Consequently, in the boundaries of ROA the value of the company will be simulated using the coefficients of annualized standard deviation for each type of production. For the BOPM a standard deviation is required to estimate jump factors causing the value of an underlying to move either up or down. Based on Jarrow-Rudd's expressions in (2.3.12), the following factors for upside and downside movements are determined in Table 6.

**Table 6. Jump factors and risk-neutral probability for diverse volatility rates**

| **Volatility** | **23%** | **29.50%** | **37%** |
|---|---|---|---|
| **Upside factor** | 1.311341 | 1.375734 | 1.44636 |
| **Downside factor** | 0.827828 | 0.762607 | 0.690078 |
| **Risk-neutral probability (p)** | 0.5005088 | 0.5010760 | 0.5021303 |
| **(1-p)** | 0.4994912 | 0.4989240 | 0.4978697 |

Another element that matters for analysis is growth rate (drift). Similar to the case with volatility, different sets of prices deliver different values of drift. However, in the option valuation model used in this paper the actual rates of growth are replaced with a risk-free rate. Still, the rate is required to estimate the total present value of a company. But rather than using all the three rates available, it is decided to select only 3% as a working variable.

In the next section, the results presented will be analysed and discussed using the framework of the BOPM. In terms of structure, it means that the set of possible scenarios obtained from conventional NPV analysis will be broadened such that each outcome will be exposed to the different rates of volatility yielding ultimately 27 variants of the model. Again, it should be added that each variant includes three binomial lattices visualizing the value of the project with two options implemented separately and the same two options included together.

# 5. Analysis and discussion

## 5.1 Explanatory comments

The intention of this chapter is to address the research questions raised at the beginning of the paper. But prior to that, a few additional clarifications should be made to ease understanding of some concepts used in the model of option valuation.

In the previous section, the results of conventional NPV analysis performed in respect to different tax scenarios with a use of several rates of discount were introduced. As frequently noticed in financial theory, this procedure fails to provide a comprehensive estimation of the profitability of an investment project if a significant uncertainty takes place in a market environment. The only parameter utilized to account for the riskness of investments is discount rate. However, this rate allows to exclusively consider the influence of time that erodes the value of a project exponentially. Still, future economic benefits subject to the time factor are presumed to stay constant implying that the intrinsic dynamics of a market is simply ignored (Brealey and Myers, 2003).

The binomial option pricing model introduced in the theoretical part enables to eliminate this disadvantage letting the value of a project float due to its exposure to the market volatility. Additionally, this model does acknowledge the decisive power of a decision-maker who is able to impact the course of investments in response to the market situation. The factor of a decision-maker aimed at maximizing economic benefits and minimizing losses is given in the form of different options, such as waiting to invest, expansion, mothballing, abandonment etc. Being implemented, these initiatives may radically change the value of a project and question the results of conventional NPV analysis.

For the purpose of this paper, it is decided to examine both option to expand and option to abandon as such that better describe the plans of the leadership of the investigated company, as well as the market environment associated with the investment project. The introduction of the options is based on the framework by Koller *et al.* (2010) that was thoroughly reviewed earlier in the dissertation. Surely, in comparison to that example, the object of case study has own specifics.

*Option to expand* reflects the intention to increase the current capacities of the steel factory. After its implementation, the value of the project will double while the amount of outflows related to additional capital costs will reach $66,9 mln. To figure out whether the expansion is worth undertaking or not, the present value of the company should be multiplied by the factor of two, and the obtained result then should be reduced by 66,9. By doing so, it is assumed that the realization of the investments in the added capacities does not take longer than a period. Moreover, being injected, this capital provides an immediate positive effect in terms of cash inflows. In a binomial lattice, this procedure is repeated at each node except for the node at year 0 as that time the first stage of the project was realized. According to the leadership's assumption, it is unreasonable to commence both stages of the project simultaneously since the pilot investments are made to test the market environment. Therefore, the opportunity to expand can be kept open as long as the market conjuncture turns favorable enough (Pers.Com., Risukhin, 2012). It is also implied that the value of investment capital in expansion remains constant during the whole period covered by the lattice.

*Option to abandon* is implemented when the value of the company becomes lower than a certain critical threshold. In literature, this threshold is usually associated with a price at which fixed assets can be sold in the market. Quite often this value coincides with a balance-sheet value of fixed assets net of the amount of depreciation, but it also can be higher or lower depending on the circumstances under which the selling occurs (Brealey and Myers, 2003). In case of Metals and Polymers, shutting down becomes more profitable than carrying on when the present value of the company drops below the level of the amount of capital invested in kit and infrastructure during the pilot stage. From Appendix 7 it is clear that this value amounts to a bit more than \$65 mln. The important notice is that the chosen abandonment value does not get reduced due to the depreciation procedure applied. According to chief shareholder interviewed for this paper, this implication relates to the fact that the legally enforced requirement to depreciate fixed capital results in a lower value of taxable income subject to the imposition of a corporate levy. As a consequence, a part of income exempt from taxation is reinvested back into maintaining and renovating fixed capital that allows to keep its value intact (Pers.Com., Risukhin, 2012). Ultimately, at each node of a binomial lattice, where the abandonment option is introduced, the company gets a lump-sum compensation equalling \$65 mln. All the nodes situated lower than those with the abandonment option become inactive.

By form, a model of a binomial lattice constructed for this paper includes 66 nodes that enables to track the evolution of the company's value in a 11-year perspective (including year 0). For each combination of independent variables, there are three versions of the lattice: in the presence of expansion option only, in the presence of abandonment option only, and in the presence of both options implemented together. The idea to analyse the options in such a way is to check the assumption by Trigeorgis (1993) that the sum of the options introduced separately exceeds the sum of the same options implemented simultaneously. For this paper, a special terminology is applied to clearly distinguish between these two sums. The *aggregated value* is referred to the sum of separated options, while the *integrated value* points to the sum obtained when the options are merged.

It is worth putting down again two research questions raised in this paper:

1) How will the options to expand and to abandon affect the present value of the analyzed company if both of them are implemented separately and together?

2) How will the value of the analyzed company and the values of the embedded options change in response to changes in key variables, such as rate of corporate tax, discount rate and volatility?

In order to address them, tables 7, 8 and 9 were set up. They aggregate the results of extended NPV analysis with respect to independent variables selected for simulation. In contrast to conventional NPV analysis (Table 4), the value of the company is adjusted by embedding opportunities to expand and to abandon. Moreover, the valuation process is carried out in the presence of several volatility rates. Key dependant variables here are extended present values and extended NPVs. Extended present values are derived by summing up the present values without options with the values of options. Two versions of extended present value are calculated by considering either the value of abandonment option or the value of expansion option. The other two versions of extended present value show the present value of the project summed up either with integrated or aggregated value. All the versions of extended NPV are derived by subtracting the value of investments from the corresponding extended present values.

## 5.2 The impact of real options on the company's value

A deeper look into the tables strongly convinces that the real options embedded in the model lead to a significant increase in the value of the company. It can be noticed that together the options account for almost a half of the extended present value irrespective of changes in tax policy, discount rate or market volatility. In the situation with lower discount rates this fact does not make a difference since even in the absence of options the amount of proceeds is so high that the investments are covered with a big excess. However, when discount rate is the highest assumed, the options count a lot as due to them the project becomes profitable, which is in a big contrast to the results of conventional NPV analysis.

Table 4 helps to remind that in the absence of options, the project exposed to the highest uncertainty (discount rate 13%) should be rejected as its NPV is deeply negative regardless of taxation policy. When the managerial flexibility is assumed, the negative effect of the highest discount rate is mitigated or even completely set off leading to the revision of the verdict based on the conventional NPV. For instance, in the lower tax scenario with discount rate 13% and volatility 37% (Table 7) even implementing abandonment option alone makes the project profitable. In a harsher environment (the same discount rate, tax rate 25%), only the combination of two options adds enough value to justify the project realization (Table 8). However, in the worst possible scenario with both tax rate and discount rate selected the highest and low coefficients of volatility, the real options are powerless to make the project worth undertaking (Table 9). But still, its financial prospects in terms of extended NPV look much brighter, and embedding additional options may even improve them.

**Table 7. The results of extended NPV analysis for scenario lower tax (tax rate 20%)**

| | Discount rate 6.75% | | | Discount rate 9.5% | | | Discount rate 13% | | |
|---|---|---|---|---|---|---|---|---|---|
| **Volatility** | **23%** | **29.50%** | **37%** | **23%** | **29.50%** | **37%** | **23%** | **29.50%** | **37%** |
| Present value (PV) | 269,865.9 | 269,865.9 | 269,865.9 | 126,214.3 | 126,214.3 | 126,214.3 | 57,046.3 | 57,046.3 | 57,046.3 |
| Integrated value | 235,829.3 | 236,544.9 | 238,882.3 | 93,459.1 | 96,836.2 | 102,107.2 | 34,354.2 | 40,029.2 | 47,566.4 |
| Aggregated value | 235,829.3 | 236,563.8 | 238,932.4 | 93,572.4 | 97,171.0 | 102,561.6 | 37,999.7 | 43,251.7 | 50,147.1 |
| Expansion option | 235,810.8 | 236,167.0 | 237,253.6 | 92,763.4 | 94,334.5 | 96,630.0 | 27,523.8 | 29,918.0 | 33,247.2 |
| Abandonment option | 18.5 | 396.8 | 1,678.9 | 809.0 | 2,836.5 | 5,931.7 | 10,475.9 | 13,333.7 | 16,899.9 |
| Ext. PV (expansion) | 505,676.7 | 506,032.9 | 507,119.5 | 218,977.6 | 220,548.7 | 222,844.2 | 84,570.1 | 86,964.3 | 90,293.5 |
| Ext. PV (abandonment) | 269,884.4 | 270,262.7 | 271,544.8 | 127,023.3 | 129,050.8 | 132,145.9 | 67,522.2 | 70,380.0 | 73,946.1 |
| Ext. PV (integrated) | 505,695.2 | 506,410.8 | 508,748.2 | 219,673.4 | 223,050.4 | 228,321.4 | 91,400.5 | 97,075.5 | 104,612.7 |
| Ext. PV (aggregated) | 505,695.2 | 506,429.7 | 508,798.4 | 219,786.6 | 223,385.2 | 228,775.9 | 95,046.0 | 100,298.0 | 107,193.4 |
| Ext. NPV (expansion) | 433,377.5 | 433,733.8 | 434,820.3 | 146,678.5 | 148,249.6 | 150,545.1 | 12,270.9 | 14,665.2 | 17,994.3 |
| Ext. NPV (abandonment) | 197,585.2 | 197,963.6 | 199,245.6 | 54,724.1 | 56,751.6 | 59,846.8 | - 4,776.9 | - 1,919.2 | 1,647.0 |
| Ext. NPV (integrated) | 433,396.0 | 434,111.6 | 436,449.0 | 147,374.2 | 150,751.3 | 156,022.3 | 19,101.3 | 24,776.4 | 32,313.5 |
| Ext. NPV (aggregated) | 433,396.0 | 434,130.6 | 436,499.2 | 147,487.5 | 151,086.1 | 156,476.7 | 22,746.8 | 27,998.8 | 34,894.2 |
| Investments | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 |

Implemented individually, the real options improve the company's profitability, but the overall effect is much weaker. Again, in favorable scenarios, with different tax rates, but the lowest and the average discount rates, the options clearly boost the value of the project, but their importance is not that significant considering the fact that decision-makers are not compelled to make a choice whether to commence or not. As a consequence, the way of implementation does not matter a lot. Additionally, it is evident that the benefits from abandonment are so minor that ignoring it in the calculation of the extended NPV will definitely not change the fate of the company. The totally opposite is true in less favorable scenarios. As the present value decreases because of the application of higher discount rates, the resources of the company to reimburse the investment spending get exhausted and the likelihood of shutting the capacities down gets higher. In the wake of this, the significance of capitalizing on the opportunity to abandon becomes noticeable in the extended NPV value . Especially it can be observed in the base case scenario with discount rate 13% where the both options are practically in complete parity in terms of their contribution to the overall value. Furthermore, in the most dramatic case (tax rate 30% and discount rate 13%), where the perspective of phasing the factory out is almost inescapable, the value of abandonment option exceeds the value of expansion option.

These findings are in a perfect coherence with financial theory. Expansion option is a type of a call option which is directly contingent on the value of an underlying. That is, an increase in the value of an underlying results in a corresponding increase in the value of option. The opposite dependence happens between the value of an underlying and abandonment option which belongs to the family of put options. A raise in the value of an underlying reduces the likelihood of a put option being exercised. Therefore, the value of a put shrinks (Copeland and Antikarov, 2001). The thesis of Trigeorgis (1993) that the aggregated value of implemented options exceeds their integrated value also generally holds in the given model. However, it should be pointed out that this effect, known as non-additivity, only relates to the combination between put and call, while other models may have own properties. For instance, a fusion of several calls is likely to lead to the effect of super-additivity, meaning that their simultaneous implementation yields value, which is higher than that obtained from a simple summation of the values of the same calls introduced separately. But, in fact, this observation bears rather a theoretical interest, because in a real-world setting investment projects once initiated cannot be reversed back to some starting point to test the impact of different managerial opportunities from scratch.

**Table 8. The results of extended NPV analysis for base case scenario (tax rate 25%)**

| | Discount rate 6.75% | | | Discount rate 9.5% | | | Discount rate 13% | | |
|---|---|---|---|---|---|---|---|---|---|
| **Volatility** | **23%** | **29.50%** | **37%** | **23%** | **29.50%** | **37%** | **23%** | **29.50%** | **37%** |
| Present value | 241,770.3 | 241,770.3 | 241,770.3 | 109,260.2 | 109,260.2 | 109,260.2 | 45,488.3 | 45,488.3 | 45,488.3 |
| Integrated value | 207,804.2 | 208,373.1 | 211,689.2 | 77,133.9 | 81,051.8 | 87,543.9 | 29,133.9 | 35,332.4 | 42,020.4 |
| Aggregated value | 207,805.1 | 208,778.2 | 211,801.7 | 77,354.5 | 81,576.2 | 88,361.6 | 34,667.1 | 40,041.9 | 46,194.5 |
| Expansion option | 207,750.4 | 208,223.1 | 209,602.7 | 76,050.2 | 77,863.7 | 80,773.7 | 17,636.6 | 20,678.0 | 23,872.0 |
| Abandonment option | 54.6 | 555.0 | 2,199.0 | 1,304.2 | 3,712.5 | 7,587.9 | 17,030.5 | 19,363.9 | 22,322.4 |
| Ext. PV (expansion) | 449,520.7 | 449,993.4 | 451,373.0 | 185,310.4 | 187,123.8 | 190,033.9 | 63,124.9 | 66,166.3 | 69,360.3 |
| Ext. PV (abandonment) | 241,824.9 | 242,325.3 | 243,969.2 | 110,564.4 | 112,972.7 | 116,848.1 | 62,518.8 | 64,852.2 | 67,810.7 |
| Ext. PV (integrated) | 449,574.5 | 450,143.4 | 453,459.5 | 186,394.1 | 190,312.0 | 196,804.1 | 74,622.2 | 80,820.7 | 87,508.6 |
| Ext. PV (aggregated) | 449,575.4 | 450,548.4 | 453,571.9 | 186,614.6 | 190,836.3 | 197,621.8 | 80,155.4 | 85,530.2 | 91,682.7 |
| Ext. NPV (expansion) | 377,221.6 | 377,694.2 | 379,073.8 | 113,011.2 | 114,824.7 | 117,734.7 | - 9,174.3 | - 6,132.8 | - 2,938.9 |
| Ext. NPV (abandonment) | 169,525.8 | 170,026.2 | 171,670.1 | 38,265.2 | 40,673.5 | 44,548.9 | - 9,780.4 | - 7,447.0 | - 4,488.4 |
| Ext. NPV (integrated) | 377,275.3 | 377,844.2 | 381,160.4 | 114,095.0 | 118,012.8 | 124,504.9 | 2,323.0 | 8,521.5 | 15,209.5 |
| Ext. NPV (aggregated) | 377,276.2 | 378,249.3 | 381,272.8 | 114,315.5 | 118,537.2 | 125,322.6 | 7,856.2 | 13,231.0 | 19,383.6 |
| Investments | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 |

What is really important for a decision-maker is that the investment project with several options assumed will certainly be more profitable than that with no option on agenda.

## 5.3 The significance of changes in independent variables

There are also some exceptions that stand out from the rest of results. The first one applies to the lower tax scenario with discount rate 6.75% and volatility 23% where a perfect identity between the integrated and aggregated values of options is observed (Table 7). That is, the effect of non-additivity as a result of integration of options of different types does not exist. In principle, Trigeorgis (1993) does not specify whether this situation is possible or not. But again, it has no practical implication in terms of profitability of the analyzed project as accepting options is always better than denying them. The second exception relates to the higher tax scenario with discount rate 13% and volatility 23% (Table 9). Here the determination of the aggregated value is complicated as the abandonment option itself cannot be properly evaluated based on the binomial lattice constructed. The reason is that the project is so deeply out-of-the-money that the evaluation procedure using backward induction interrupts too early without reaching the starting node at year 0. At the same time, being introduced in a bunch with expansion option, the opportunity to abandon still can get estimated. That is, the value of the abandonment option can be found indirectly by subtracting the value of expansion in separation from the integrated value. However, this figure equaling $20,547.88 mln is just an approximation.

**Table 9. The results of extended NPV analysis for scenario higher tax (tax rate 30%)**

| | Discount rate 6.75% | | | Discount rate 9.5% | | | Discount rate 13% | | |
|---|---|---|---|---|---|---|---|---|---|
| **Volatility** | **23%** | **29.50%** | **37%** | **23%** | **29.50%** | **37%** | **23%** | **29.50%** | **37%** |
| Present value | 213,674.7 | 213,674.7 | 213,674.7 | 92,306.1 | 92,306.1 | 92,306.1 | 33,930.2 | 33,930.2 | 33,930.2 |
| Integrated value | 179,779.1 | 180,982.2 | 184,496.2 | 61,841.8 | 66,070.7 | 73,116.4 | 30,274.7 | 34,287.7 | 38,793.8 |
| Aggregated value | 179,785.2 | 181,047.0 | 184,674.8 | 62,312.8 | 66,897.3 | 74,219.4 | - | 39,114.1 | 43,417.8 |
| Expansion option | 179,690.1 | 180,279.2 | 181,951.8 | 59,865.7 | 61,589.8 | 64,917.5 | 9,726.8 | 12,136.9 | 14,496.8 |
| Abandonment option | 95.2 | 767.8 | 2,723.0 | 2,447.1 | 5,307.5 | 9,302.0 | - | 26,977.2 | 28,921.0 |
| Ext. PV (expansion) | 393,364.7 | 393,953.9 | 395,626.5 | 152,171.8 | 153,895.9 | 157,223.5 | 43,657.0 | 46,067.2 | 48,427.1 |
| Ext. PV (abandonment) | 213,769.8 | 214,442.4 | 216,397.7 | 94,753.2 | 97,613.5 | 101,608.0 | - | 60,907.4 | 62,851.2 |
| Ext. PV (integrated) | 393,453.7 | 394,656.9 | 398,170.9 | 154,147.9 | 158,376.8 | 165,422.4 | 64,204.9 | 68,217.9 | 72,724.1 |
| Ext. PV (aggregated) | 393,459.9 | 394,721.7 | 398,349.5 | 154,618.9 | 159,203.3 | 166,525.5 | - | 73,044.3 | 77,348.0 |
| Ext. NPV (expansion) | 321,065.6 | 321,654.7 | 323,327.3 | 79,872.6 | 81,596.7 | 84,924.4 | -28,642.1 | - 26,232.0 | - 23,872.1 |
| Ext. NPV (abandonment) | 141,470.7 | 142,143.3 | 144,098.5 | 22,454.0 | 25,314.4 | 29,308.9 | - | - 11,391.8 | - 9,447.9 |
| Ext. NPV (integrated) | 321,154.6 | 322,357.7 | 325,871.7 | 81,848.7 | 86,077.6 | 93,123.3 | - 8,094.3 | - 4,081.3 | 424.9 |
| Ext. NPV (aggregated) | 321,160.8 | 322,422.5 | 326,050.3 | 82,319.8 | 86,904.2 | 94,226.3 | - | 745.2 | 5,048.9 |
| Investments | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 | 72,299.2 |

Generally, all the independent variables subject to simulation affect the value of the company and the options implemented, but the character of this influence varies depending on where those variables are situated in the chain of analysis.

From Appendix 7 and Tables 7-9 it is seen that cash flows should be first exposed to rate of corporate tax in order to become eligible for further simulation. The higher the percent of imposition, the lower the value of both the company and the expansion option is set to be. On the contrary, the value of the abandonment option increases since the burdensome tax rate makes the company less profitable and more exposed to market perturbations. Still, there is one important factor to apply that can either enhance the result of taxation policy, or neutralize it. For instance, the base case scenario is represented by tax rate 25% and discount rate 9,5% (Table 8). If taxation policy becomes stricter, i.e. rate of corporate levy increases to 30%, the value of cash flows subject to discounting declines. Still, the impact caused by the change in tax rate can be well compensated should discount rate decrease to 6,75% (Table 9). Surely, the result can even worsen if the both independent variables increase at the same time leading to the less desirable scenario in the right side of Table 9.

The rate of volatility is applied to the present value of cash flows (discounted cash flows) and directly affect the value of the dependent variables. As the rate of volatility increases, so does the value of both options, because the distribution of possible outcomes under the greater uncertainty is wider. It means that the expansion of production capacities, as well as the abandonment procedure, can be carried out earlier. Also, as pointed out before, the rate of volatility makes an economic sense if it is considered in the context of options introduced. Because of options, the present value of the business increases while the volatility makes this increase bigger or smaller. Therefore, there is no reason to discuss whether volatility may correct the impact of taxation policy or discount rate. At the same time, its rate can be crucial for projects with negative conventional NPV. Returning to Table 9 (discount rate 13%), it is quite visible that the project remains unprofitable with volatility rate 29,5%, but it is worth realizing if volatility grows to 37%. Here it is also interesting to observe that with volatility 29,5%, aggregated NPV is positive and the project is profitable, while integrated NPV still stays negative. As agreed earlier, a decision-maker should focus on the integrated value as such that better reflects reality.

## 5.4. Possible aspects of future studies

The current paper managed to address some important aspects of financial theory that can be of an interest for decision-makers. At the same time, administrative requirements imposed, as well as succinctness of empirical data and time factor put some limitations on the research process. There are several problems worth considering to enrich and complement the study in future.

Within a theoretical model presented, a few independent variables were deemed to be deterministic to emphasize the impact of changes in stochastic variables selected. Letting more variables float will certainly make the model more sophisticated, but still will increase the range of scenarios to be taken into account by a decision-maker. For instance, the coefficient of cost-price squeeze can be loosen to account for the additional uncertainty associated with the amount of margin between input and output prices. For instance, a set of three values of cost-price squeeze will triple the number of scenarios in the model, provided the other stochastic and deterministic variables are kept intact.

Credibility of the analysis also can be improved by modifying the way how some variables are considered in the model. For instance, the amount of investments, applied in the evaluation of the expansion option, can be subject to discounting to demonstrate the consequences of the decision to expand being postponed. Currently, it is assumed that expansion can be delayed so long as waiting is better than exercising the expansion option immediately. Since the expansion option implemented in the paper is a type of American call option with no dividends, premature exercise is always unreasonable. However, if the value of the investments gets reduced in time due to discounting, the value of expansion option raises, and expansion at some nodes preceding maturity may be more rewarding than waiting. Still, this implication is not certain and should be carefully inspected in practice. By analogy, the value of the abandonment option can be adjusted too by including in calculation possible expenditures associated with an abandonment procedure. These expenditures are cash outflows that reduce the amount of gain to be obtained from selling fixed assets out. Under such circumstances, abandonment may be postponed as keeping the project alive may be still more profitable than phasing out.

Implementing other options seems logical as a continuation of the current analysis, but in terms of the given project there are some restrictions that should be taken into account. For instance, an option to switch is a reliable way to hedge against uncertainty in both input and output. Practically, it is a flexibility that allows a business to manoeuvre in a market by choosing either a raw material to use in production process or a type of commodity to manufacture. A well-known example of this is the electricity generator capable of producing power from different types of fuel, usually gas and coal. Depending on the price situation, the equipment is engineered to switch to that type of fuel which is cheaper (Copeland and Antikarov, 2003). Regarding Metals and Polymers, the option to switch is embedded in its production capacities with two production lines enabling to manufacture both galvanized steel and polymerized steel. As highlighted previously, these lines are technologically separated systems so that each one can be suspended without causing the idleness of another one. Consequently, hinging on demand and price fluctuations, the company is capable to switch to either output (Pers.Com., Risukhin, 2012). However, evaluation of the opportunity to switch requires more data from financial statements that is not available. In particular, estimations of demand and price, revenue and cost of revenue, and operating expenses should be provided. This information is needed to figure out to what extent each of the production lines contributes to the company's overall value. Appendix 7, which is the only source of financial data about the analysed company, does not contain these details.

The important dimension of analysis that was sacrificed for the sake of simplicity is opportunity costs incurred due to the postponement of expansion option. It is evident that once exercised the option to expand leads to generation of additional cash inflows. If the option to expand is worth exercising, but a decision-maker decides to delay in expectation of brighter economic prospects, she is supposed to account for those inflows as lost profits or, in other words, opportunity costs. In financial literature, opportunity costs are tantamount to dividends paid on a stock which are difficult to consider by means of the two-state binomial model. The major complication lays in the structure of the lattice that radically changes after the ex-dividend date, i.e. a point of time when the dividends are paid out (Schroder, 1988).

Visually it means that in parallel to already existent lattice a new binomial tree should be built up based on the values derived after the sum of dividends is subtracted. In the presence of the only ex-dividend date this difficulty may not seem bearable, but if postponement happens for several periods in a raw, which is often probable scenario, both the visualization and valuation processes become tremendously complicated (Brandao *et al.*, 2005).

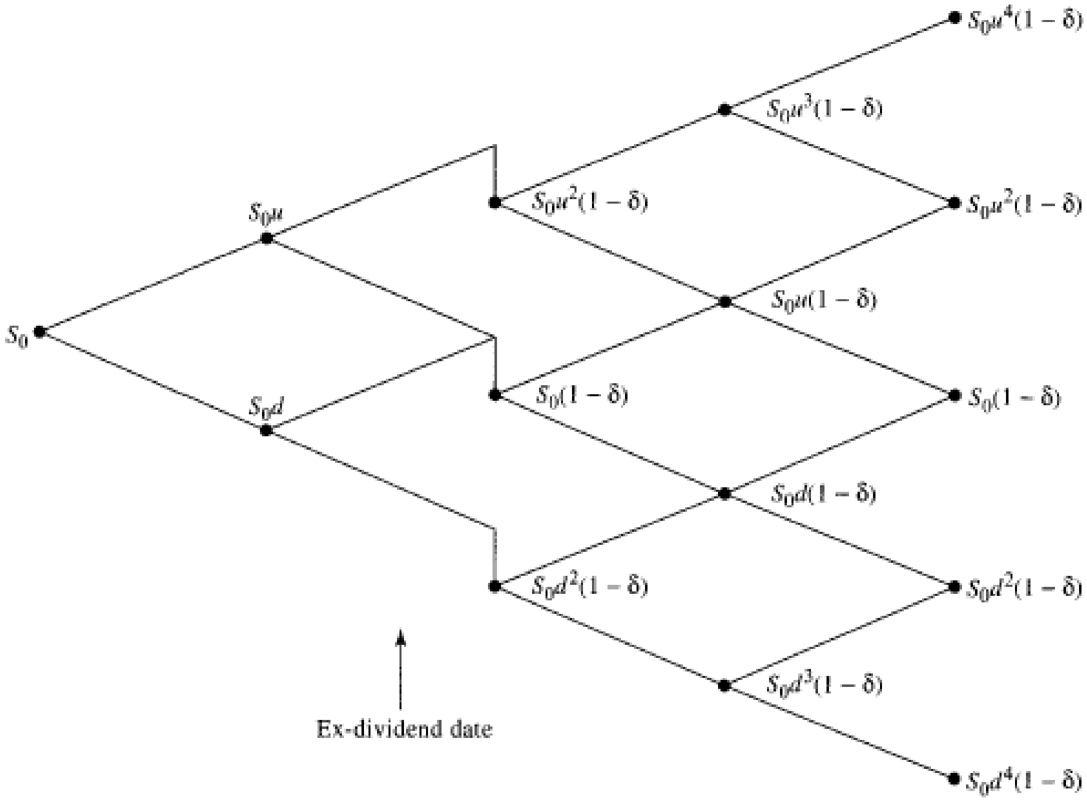


**Fig 6. The impact of dividends on the structure of a binomial tree** (Hull, 2000)

Figure 6 illustrates the change in the structure of the binomial lattice subject to one-time dividends with a fixed percent yield. Here, $S_o$ is the value of an underlying, $u$ and $d$ are jump factors, and $\delta$ is a dividend yield. In respect to the analysed project, a scenario with the only ex-dividend date can be employed by the way of experiment. In addition, as a more feasible alternative, a model of trinomial lattice can be consulted. In contrast to the two-state binomial model, assuming the value of an underlying changes either up or down, the trinomial lattice retains these two opposite states and considers a third one implying no change occurs. As can be expected, at the expense of this additional element, the number of outcomes will increase and that will also make the structure of the lattice more sophisticated (Boyle, 1986).

# 6. Conclusion

The current work is dedicated to investigating the influence of real options on the value of a business entity. Real options are described as different business opportunities that decision-makers are encouraged to utilize in order to maximize profits and reduce losses. The taxonomy of real options is vast, but every investment project has own properties that dictate the choice of options applicable. Analysis in this paper revolves around the case of steel-making company Metals and Polymers situated in Ukraine. The company is a brand-new investment project established in quite a risky environment, but has ambitions of development through expansion of current capacities. Based on financial data, provided by the company's leadership, it is possible to pick up two managerial opportunities that can get examined by means of real options theory, namely option to expand and option to abandon.

The binomial option pricing model represents a popular estimation tool thanks to its intuitive design allowing to visually demonstrate the evolution of the value of a business project in the presence of real options. Mathematical background underlying the BOPM used in this dissertation can be found in books of Jarrow and Rudd (1983) and Jarrow and Turnbull (2000). Comparably to the mainstream version of BOPM proposed by Cox *et al.*(1979), this model develops by including into analysis additional element called *drift* that enhances the effect of *volatility* and makes the value of the underlying project higher. As a consequence, the impact of the real options implemented is also stronger as their values are in a functional dependence on the value of an underlying.

The results of analysis are generally consistent with the paradigms of financial theory. As expected, conventional NPV analysis that assumes no option in its framework delivers an estimate that significantly underrates the potential of the project under consideration. NPV values are almost twice as lower as those obtained in the extended version of NPV with options implemented. That is, based on the financial data available, it is clear that the opportunities to expand and abandon are very significant in terms of value added. Their presence is especially essential in cases where the investment project is deemed to be out-of-the-money by measures of conventional NPV analysis. Bolstered by options included, the company is back on the path of profitability, except for the most dramatic case with economic conditions chosen are so formidable that even the options are not able to recover its value.

The impact of independent variables exposed to simulation is also quite predictable. Higher discount rates and rates of corporate tax lead to lower present value that reduces the chances of the project to end up in-the-money. Still, the range of possible combinations between them is pretty wide and the detrimental effect of one of them can be utterly offset by the impact of another one. Volatility rate is presumed to be considered in the context of options, and therefore in isolation it is not set to modify the circumstances conditioned by discount rate and tax rate. However, the role of volatility is crucial in situation when the project is hanging on the balance of profitability, and the value added by options can cause the decision-makers to either reject or to accept the project.

Despite the research questions raised in the paper are successfully addressed, there are other aspects to consult that are able to complement and enrich the results of the current study. In particular, more independent variables, such as the cost-price squeeze factor and costs of expansion, can be involved in analysis. Other managerial opportunities, namely option to switch, can be also potentially touched, but for this more information describing the company's financial records should be revealed. An interesting avenue of research is the opportunity costs incurred because of delaying expansion. In financial theory they are treated as dividends paid out on a stock. However, their involvement is reasonable at the elementary level only (with the only ex-dividend date) since more serious complications are associated with difficulties of computation and visualization.

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## Personal communication

Risukhin, Vladimir, chief shareholder of Metals and Polymers Ltd. (interview, 2012-01-30)

## Appendix 1. The time series analysis for different types of rolled steel

| Production | | Polymerized steel | | Galvanized steel | | Cold-rolled steel | |
|---|---|---|---|---|---|---|---|
| **Date** | **t** | **Y ($/ton)** | **Ln(Yt) - Ln(Yt-1)** | **Y ($/ton)** | **Ln(Yt) - Ln(Yt-1)** | **Y ($/ton)** | **Ln(Yt) - Ln(Yt-1)** |
| Dec-10 | 48 | 1500.0 | 0 | 1075.0 | 0 | 962.5 | 0 |
| Nov-10 | 47 | 1500.0 | 0.016807118 | 1075.0 | -0.011560822 | 962.5 | 0 |
| Oct-10 | 46 | 1475.0 | 0 | 1087.5 | 0 | 962.5 | 0 |
| Sep-10 | 45 | 1475.0 | -0.049596941 | 1087.5 | -0.022728251 | 962.5 | 0 |
| Aug-10 | 44 | 1550.0 | 0 | 1112.5 | 0 | 962.5 | 0 |
| Jul-10 | 43 | 1550.0 | -0.016000341 | 1112.5 | 0 | 962.5 | 0 |
| Jun-10 | 42 | 1575.0 | -0.00790518 | 1112.5 | -0.011173301 | 962.5 | 0.013072082 |
| May-10 | 41 | 1587.5 | 0 | 1125.0 | 0 | 950.0 | 0.076540077 |
| Apr-10 | 40 | 1587.5 | 0.056695344 | 1125.0 | 0 | 880 | -0.06329485 |
| Mar-10 | 39 | 1500.0 | -0.016529302 | 1125.0 | 0.01342302 | 937.5 | 0 |
| Feb-10 | 38 | 1525.0 | 0 | 1110 | 0.061540018 | 937.5 | 0 |
| Jan-10 | 37 | 1525.0 | 0.103540679 | 1043.8 | 0 | 937.5 | 0.120286166 |
| Dec-09 | 36 | 1375 | 0.018349139 | 1043.8 | 0.012048339 | 831.3 | 0 |
| Nov-09 | 35 | 1350 | 0.028170877 | 1031 | -0.006042314 | 831 | 0.038318864 |
| Oct-09 | 34 | 1313 | 0.048790164 | 1038 | -0.052798186 | 800 | 0.015748357 |
| Sep-09 | 33 | 1250 | -0.009950331 | 1094 | -0.028170877 | 788 | -0.015748357 |
| Aug-09 | 32 | 1263 | 0.030153038 | 1125 | 0.223143551 | 800 | 0.048009219 |
| Jul-09 | 31 | 1225 | 0.057758834 | 900 | -0.034133006 | 763 | 0.04184711 |
| Jun-09 | 30 | 1156 | -0.037139547 | 931 | 0.034133006 | 731 | 0.043675064 |
| May-09 | 29 | 1200 | 0 | 900 | 0 | 700 | 0.113328685 |
| Apr-09 | 28 | 1200 | 0.021053409 | 900 | -0.013793322 | 625 | -0.076961041 |
| Mar-09 | 27 | 1175 | 0 | 913 | -0.006825965 | 675 | -0.036367644 |
| Feb-09 | 26 | 1175 | 0 | 919 | -0.046520016 | 700 | -0.026433257 |
| Jan-09 | 25 | 1175 | -0.1480531 | 963 | -0.038221213 | 719 | -0.122602322 |
| Dec-08 | 24 | 1363 | -0.087775611 | 1000 | 0.012578782 | 813 | 0.096849826 |
| Nov-08 | 23 | 1488 | -0.265591115 | 988 | 0 | 738 | -0.081345639 |
| Oct-08 | 22 | 1940 | -0.079249372 | 988 | -0.470003629 | 800 | -0.601579987 |
| Sep-08 | 21 | 2100 | 0 | 1580 | -0.043350441 | 1460 | -0.040273899 |
| Aug-08 | 20 | 2100 | 0.019231362 | 1650 | -0.053109825 | 1520 | 0 |
| Jul-08 | 19 | 2060 | 0.039609138 | 1740 | 0 | 1520 | 0 |
| Jun-08 | 18 | 1980 | 0 | 1740 | 0.047067511 | 1520 | 0.047146778 |
| May-08 | 17 | 1980 | 0.129211731 | 1660 | 0.036813973 | 1450 | 0.013889112 |
| Apr-08 | 16 | 1740 | 0.122102697 | 1600 | 0.077961541 | 1430 | 0.06500483 |
| Mar-08 | 15 | 1540 | 0.109698917 | 1480 | 0.129677823 | 1340 | 0.127155175 |
| Feb-08 | 14 | 1380 | 0.029413885 | 1300 | 0.113944259 | 1180 | 0.185717146 |
| Jan-08 | 13 | 1340 | -0.126040721 | 1160 | 0.04405999 | 980 | 0.107630664 |
| Dec-07 | 12 | 1520 | -0.013072082 | 1110 | 0 | 880 | -0.022472856 |
| Nov-07 | 11 | 1540 | 0 | 1110 | 0 | 900 | 0.011173301 |
| Oct-07 | 10 | 1540 | -0.038221213 | 1110 | -0.017857617 | 890 | -0.011173301 |
| Sep-07 | 9 | 1600 | -0.048790164 | 1130 | -0.026202372 | 900 | 0 |
| Aug-07 | 8 | 1680 | 0 | 1160 | -0.025533302 | 900 | 0 |
| Jul-07 | 7 | 1680 | 0.036367644 | 1190 | 0 | 900 | 0 |
| Jun-07 | 6 | 1620 | 0 | 1190 | 0 | 900 | 0.022472856 |
| May-07 | 5 | 1620 | 0.037740328 | 1190 | 0 | 880 | 0.034685558 |
| Apr-07 | 4 | 1560 | 0 | 1190 | 0 | 850 | 0 |
| Mar-07 | 3 | 1560 | 0.025975486 | 1190 | 0.008438869 | 850 | 0.035932009 |
| Feb-07 | 2 | 1520 | 0.026668247 | 1180 | 0.017094433 | 820 | 0 |
| Jan-07 | 1 | 1480 | | 1160 | | 820 | |
| **Average monthly st.deviation** | | | **0.066571331** | | **0.085151916** | | **0.106307738** |
| **Annualized st.deviation** | | | **0.230609856** | | **0.29497489** | | **0.368260807** |
| **Annualized mean** | | | **0.003427154** | | **-0.01942962** | | **0.040909717** |
| **Drift** | | | **0.030017607** | | **0.024075473** | | **0.108717728** |

## Appendix 2. The results of the multiple regression analysis

**POLYMERIZED STEEL**

| *Regression Statistics* | |
|---|---|
| Multiple R | 0.20964 |
| R Square | 0.04395 |
| Adjusted R Square | 0.02270 |
| Standard Error | 6.58113 |
| Observations | 47.00000 |

**ANOVA**

| | *df* | *SS* | *MS* | *F* | *Significance F* |
|---|---|---|---|---|---|
| Regression | 1 | 89.594117 | 89.594117 | 2.068610 | 0.157277 |
| Residual | 45 | 1949.007271 | 43.311273 | | |
| Total | 46 | 2038.601388 | | | |

| | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* |
|---|---|---|---|---|---|---|
| Intercept | -8.58959 | 6.06845 | -1.41545 | 0.16382 | -20.81208 | 3.63289 |
| X Variable 1 | 0.00565 | 0.00393 | 1.43827 | 0.15728 | -0.00226 | 0.01357 |

**GALVANIZED STEEL**

| *Regression Statistics* | |
|---|---|
| Multiple R | 0.22909 |
| R Square | 0.05248 |
| Adjusted R Square | 0.03143 |
| Standard Error | 0.08380 |
| Observations | 47.00000 |

**ANOVA**

| | *df* | *SS* | *MS* | *F* | *Significance F* |
|---|---|---|---|---|---|
| Regression | 1 | 0.01751 | 0.01751 | 2.49257 | 0.12139 |
| Residual | 45 | 0.31603 | 0.00702 | | |
| Total | 46 | 0.33354 | | | |

| | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* |
|---|---|---|---|---|---|---|
| Intercept | -0.10363 | 0.06576 | -1.57589 | 0.12206 | -0.23607 | 0.02882 |
| X Variable 1 | 0.00009 | 0.00006 | 1.57879 | 0.12139 | -0.00002 | 0.00020 |

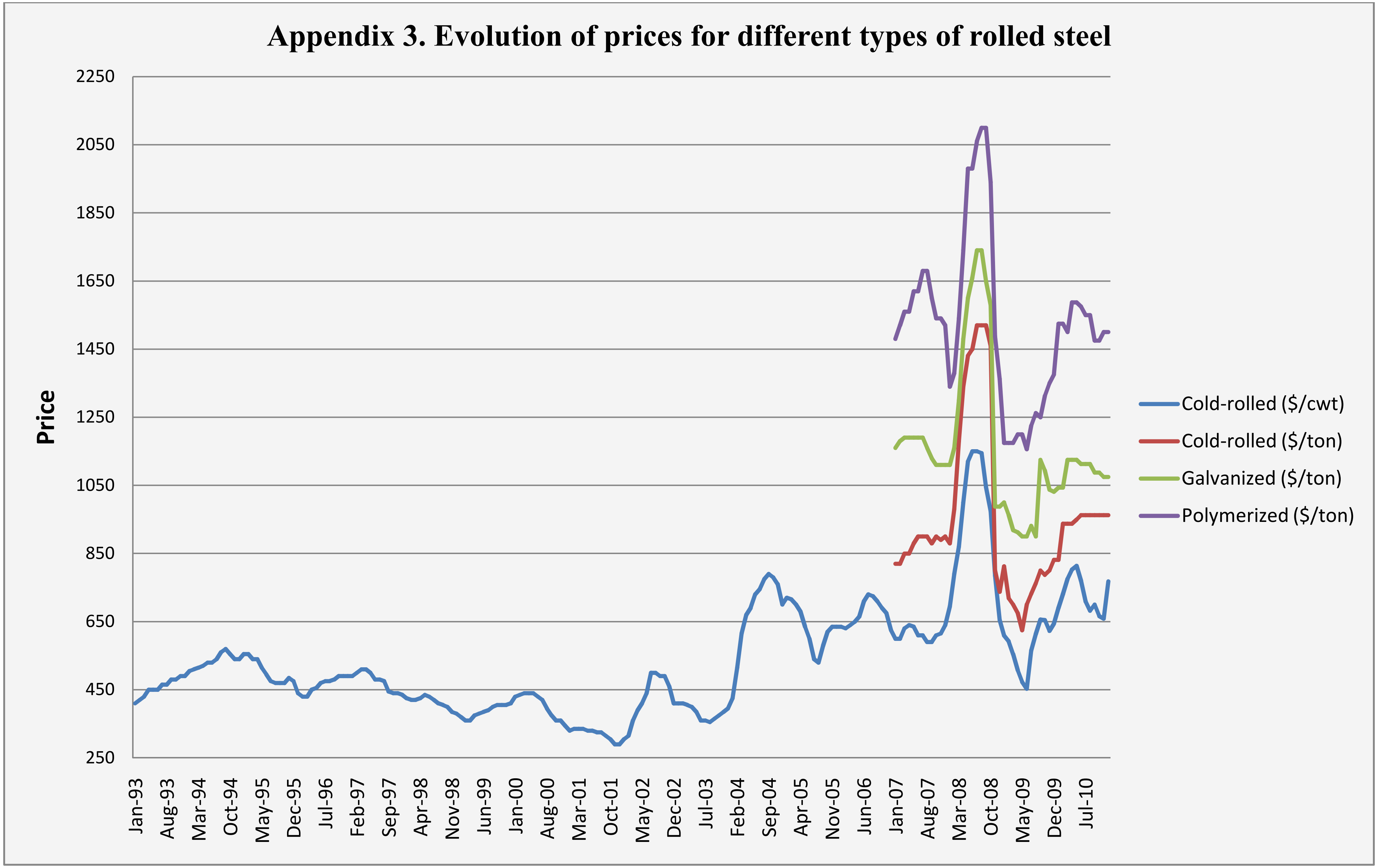
**COLD-ROLLED STEEL**

| *Regression Statistics* | |
|---|---|
| Multiple R | 0.20924 |
| R Square | 0.04378 |
| Adjusted R Square | 0.02253 |
| Standard Error | 0.10510 |
| Observations | 47.00000 |

**ANOVA**

| | *df* | *SS* | *MS* | *F* | *Significance F* |
|---|---|---|---|---|---|
| Regression | 1 | 0.02276 | 0.02276 | 2.06045 | 0.15808 |
| Residual | 45 | 0.49710 | 0.01105 | | |
| Total | 46 | 0.51986 | | | |

| | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* |
|---|---|---|---|---|---|---|
| Intercept | -0.08538 | 0.06373 | -1.33977 | 0.18705 | -0.21373 | 0.04297 |
| X Variable 1 | 0.00009 | 0.00006 | 1.43543 | 0.15808 | -0.00004 | 0.00022 |




**Appendix 3. Evolution of prices for different types of rolled steel**

**Source:** www, Steel Market Update, 2012; internal archive of Metals and Polymers

## Appendix 4. Binomial lattice with expansion option (tax rate 25%, discount rate 13%, volatility 29,5%)

| Period | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| PV | 45,488.3 | 62,579.7 | 86,093.1 | 118,441.2 | 162,943.6 | 224,167.1 | 308,394.3 | 424,268.6 | 583,680.8 | 802,989.6 | 1,104,700.1 |
| expansion | - | 58,226.4 | 105,253.1 | 169,949.3 | 258,954.1 | 381,401.0 | 549,855.5 | 781,604.0 | 1,100,428.4 | 1,539,046.0 | 2,142,467.2 |
| waiting | 66,166.3 | 95,481.9 | 137,553.6 | 197,492.1 | 282,168.2 | 400,775.0 | 565,693.3 | 793,873.8 | 1,108,881.0 | 1,543,414.9 | |
| PV | | 34,689.7 | 47,723.8 | 65,655.2 | 90,324.1 | 124,261.9 | 170,951.4 | 235,183.7 | 323,550.2 | 445,119.0 | 612,365.4 |
| expansion | | 2,446.2 | 28,514.4 | 64,377.3 | 113,715.0 | 181,590.7 | 274,969.6 | 403,434.2 | 580,167.2 | 823,304.9 | 1,157,797.7 |
| waiting | | 45,985.0 | 66,592.6 | 96,608.8 | 140,092.1 | 202,542.6 | 291,238.5 | 415,703.9 | 588,619.8 | 827,673.8 | |
| PV | | | 26,454.6 | 36,394.5 | 50,069.1 | 68,881.8 | 94,763.0 | 130,368.7 | 179,352.7 | 246,741.6 | 339,450.9 |
| expansion | | | - 14,024.0 | 5,855.8 | 33,205.1 | 70,830.4 | 122,592.9 | 193,804.3 | 291,772.3 | 426,550.1 | 611,968.6 |
| waiting | | | 31,724.7 | 45,767.3 | 66,459.4 | 96,979.7 | 141,812.4 | 206,998.4 | 300,224.8 | 430,919.0 | |
| PV | | | | 20,174.4 | 27,754.7 | 38,183.1 | 52,529.7 | 72,266.9 | 99,420.1 | 136,775.6 | 188,166.9 |
| expansion | | | | - 26,584.2 | - 11,423.8 | 9,433.0 | 38,126.3 | 77,600.7 | 131,907.0 | 206,618.1 | 309,400.6 |
| waiting | | | | 22,061.9 | 31,391.6 | 45,109.3 | 65,527.0 | 96,193.6 | 142,341.7 | 210,987.0 | |
| PV | | | | | 15,385.2 | 21,165.9 | 29,118.7 | 40,059.5 | 55,111.3 | 75,818.5 | 104,306.0 |
| expansion | | | | | - 36,162.8 | - 24,601.3 | - 8,695.8 | 13,185.9 | 43,289.4 | 84,703.8 | 141,678.9 |
| waiting | | | | | 15,779.7 | 22,008.3 | 30,917.2 | 43,899.5 | 63,309.8 | 93,322.8 | |
| PV | | | | | | 11,732.8 | 16,141.3 | 22,206.1 | 30,549.7 | 42,028.2 | 57,819.7 |
| expansion | | | | | | - 43,467.5 | - 34,650.6 | - 22,520.9 | -5,833.8 | 17,123.3 | 48,706.2 |
| waiting | | | | | | 11,732.8 | 16,141.3 | 22,206.1 | 30,549.7 | 42,028.2 | |
| PV | | | | | | | 8,947.5 | 12,309.4 | 16,934.5 | 23,297.4 | 32,051.0 |
| expansion | | | | | | | -49,038.0 | - 42,314.2 | - 33,064.1 | - 20,338.3 | - 2,831.1 |
| waiting | | | | | | | 8,947.5 | 12,309.4 | 16,934.5 | 23,297.4 | |
| PV | | | | | | | | 6,823.5 | 9,387.3 | 12,914.4 | 17,766.8 |
| expansion | | | | | | | | - 53,286.2 | - 48,158.6 | - 41,104.4 | - 31,399.6 |
| waiting | | | | | | | | 6,823.5 | 9,387.3 | 12,914.4 | |
| PV | | | | | | | | | 5,203.6 | 7,158.8 | 9,848.6 |
| expansion | | | | | | | | | - 56,525.9 | - 52,615.5 | - 47,235.9 |
| waiting | | | | | | | | | 5,203.6 | 7,158.8 | |
| PV | | | | | | | | | | 3,968.3 | 5,459.3 |
| expansion | | | | | | | | | | - 58,996.5 | - 56,014.4 |
| waiting | | | | | | | | | | 3,968.3 | |
| PV | | | | | | | | | | | 3,026.3 |
| expansion | | | | | | | | | | | - 60,880.6 |
| waiting | | | | | | | | | | | |

## Appendix 5. Binomial lattice with abandonment option (tax rate 25%, discount rate 13%, volatility 29,5%)

| **Period** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **PV/abandon** | 45,488.3 | 62,579.7 | 86,093.1 | 118,441.2 | 162,943.6 | 224,167.1 | 308,394.3 | 424,268.6 | 583,680.8 | 802,989.6 | 1,104,700.1 |
| **waiting** | 64,852.2 | 73,707.7 | 91,936.6 | 121,062.3 | 163,823.9 | 224,326.3 | 308,394.3 | 424,268.6 | 583,680.8 | 802,989.6 | |
| **PV/abandon** | | **65,035.3** | 47,723.8 | 65,655.2 | 90,324.1 | 124,261.9 | 170,951.4 | 235,183.7 | 323,550.2 | 445,119.0 | 612,365.4 |
| **waiting** | | | 65,716.4 | 75,552.9 | 95,060.3 | 125,989.6 | 171,292.7 | 235,183.7 | 323,550.2 | 445,119.0 | |
| **PV/abandon** | | | **NE** | **65,035.3** | 50,069.1 | 68,881.8 | 94,763.0 | 130,368.7 | 179,352.7 | 246,741.6 | 339,450.9 |
| **waiting** | | | | | 66,535.8 | 77,302.4 | 98,124.9 | 131,100.6 | 179,352.7 | 246,741.6 | |
| **PV/abandon** | | | | **NE** | **NE** | **65,035.3** | 52,529.7 | 72,266.9 | 99,420.1 | 136,775.6 | 188,166.9 |
| **waiting** | | | | | | | 67,209.5 | 78,740.8 | 100,989.4 | 136,775.6 | |
| **PV/abandon** | | | | | **NE** | **NE** | **NE** | **65,035.3** | 55,111.3 | 75,818.5 | 104,306.0 |
| **waiting** | | | | | | | | | 67,416.9 | 79,183.5 | |
| **PV/abandon** | | | | | | **NE** | **NE** | **NE** | **NE** | **65,035.3** | **65,035.3** |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | **NE** | **NE** | **NE** | **NE** | **NE** |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | | **NE** | **NE** | **NE** | **NE** |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | | | **NE** | **NE** | **NE** |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | | | | **NE** | **NE** |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | | | | | **NE** |
| **waiting** | | | | | | | | | | | |

## Appendix 6. Binomial lattice with options to expand and abandon (tax rate 25%, discount rate 13%, volatility 29,5%)

| Period | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **PV/abandon** | 45,488.3 | 62,579.7 | 86,093.1 | 118,441.2 | 162,943.6 | 224,167.1 | 308,394.3 | 424,268.6 | 583,680.8 | 802,989.6 | 1,104,700.1 |
| **expansion** | | 58,226.4 | 105,253.1 | 169,949.3 | 258,954.1 | 381,401.0 | 549,855.5 | 781,604.0 | 1,100,428.4 | 1,539,046.0 | 2,142,467.2 |
| **waiting** | 80,820.7 | 104,598.9 | 142,707.4 | 199,965.1 | 283,048.5 | 400,934.1 | 565,693.3 | 793,873.8 | 1,108,881.0 | 1,543,414.9 | |
| **PV/abandon** | | 34,689.7 | 47,723.8 | 65,655.2 | 90,324.1 | 124,261.9 | 170,951.4 | 235,183.7 | 323,550.2 | 445,119.0 | 612,365.4 |
| **expansion** | | 2,446.2 | 28,514.4 | 64,377.3 | 113,715.0 | 181,590.7 | 274,969.6 | 403,434.2 | 580,167.2 | 823,304.9 | 1,157,797.7 |
| **waiting** | | 68,251.7 | 80,965.9 | 105,176.3 | 144,510.9 | 204,270.3 | 291,579.8 | 415,703.9 | 588,619.8 | 827,673.8 | |
| **PV/abandon** | | | **65,035.3** | 36,394.5 | 50,069.1 | 68,881.8 | 94,763.0 | 130,368.7 | 179,352.7 | 246,741.6 | 339,450.9 |
| **expansion** | | | | 5,855.8 | 33,205.1 | 70,830.4 | 122,592.9 | 193,804.3 | 291,772.3 | 426,550.1 | 611,968.6 |
| **waiting** | | | | 67,983.2 | 80,392.6 | 104,719.7 | 145,174.3 | 207,730.3 | 300,224.8 | 430,919.0 | |
| **PV/abandon** | | | | **NE** | **65,035.3** | 38,183.1 | 52,529.7 | 72,266.9 | 99,420.1 | 136,775.6 | 188,166.9 |
| **expansion** | | | | | | 9,433.0 | 38,126.3 | 77,600.7 | 131,907.0 | 206,618.1 | 309,400.6 |
| **waiting** | | | | | | 67,212.6 | 78,747.3 | 102,667.4 | 143,911.0 | 210,987.0 | |
| **PV/abandon** | | | | | **NE** | **NE** | **65,035.3** | 40,059.5 | 55,111.3 | 75,818.5 | 104,306.0 |
| **expansion** | | | | | | | | 13,185.9 | 43,289.4 | 84,703.8 | 141,678.9 |
| **waiting** | | | | | | | | 65,745.7 | 75,615.4 | 96,687.8 | |
| **PV/abandon** | | | | | | **NE** | **NE** | **NE** | **65,035.3** | **65,035.3** | **65,035.3** |
| **expansion** | | | | | | | | | | | |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | **NE** | **NE** | **NE** | **NE** | **NE** |
| **expansion** | | | | | | | | | | | |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | | **NE** | **NE** | **NE** | **NE** |
| **expansion** | | | | | | | | | | | |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | | | **NE** | **NE** | **NE** |
| **expansion** | | | | | | | | | | | |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | | | | **NE** | **NE** |
| **expansion** | | | | | | | | | | | |
| **waiting** | | | | | | | | | | | |
| **PV/abandon** | | | | | | | | | | | **NE** |
| **expansion** | | | | | | | | | | | |
| **waiting** | | | | | | | | | | | |

## Appendix 7. Free cash flow forecast in regard to different tax rates and discount rates

| Periods of activity | % rates | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 2010/11 | 2011/12 | 2012/13 | 2013/14 | 2014/15 | 2015/16 | 2016/17 | 2017/18 | 2018/19 | 2019/20 | 2020/21 | Horizon |
| **EBITDA** | | **11,568** | **26,229** | **26,229** | **26,229** | **26,229** | **26,229** | **26,229** | **26,229** | **26,229** | **26,229** | **26,229** | **26,229** |
| EBITDA (net of Cost price squeeze) | **1.5%** | 11,568 | 25,836 | 25,448 | 25,066 | 24,690 | 24,320 | 23,955 | 23,596 | 23,242 | 22,893 | 22,550 | 22,212 |
| Interest | | 6,230 | 10,396 | 11,493 | 8,708 | 5,252 | 1,450 | 0 | 0 | 0 | 0 | 0 | |
| **INVESTMENTS (phase 1)** | | | | | | | | | | | | | |
| Kit, Infrastructure | | 65,035 | | | | | | | | | | | |
| Working Capital | | 7,264 | | | | | | | | | | | |
| **Total investment** | | **72,299** | | | | | | | | | | | |
| **CASH FLOWS** | | -66,961 | 15,440 | 13,955 | 16,358 | 19,438 | 22,870 | 23,955 | 23,596 | 23,242 | 22,893 | 22,550 | 22,212 |
| | | | | | | | | | | | | | |
| **CORPORATE TAX ON EBITDA** | **25%** | 2,892.00 | 6,557.25 | 6,557.25 | 6,557.25 | 6,557.25 | 6,557.25 | 6,557.25 | 6,557.25 | 6,557.25 | 6,557.25 | 6,557.25 | 6,557.25 |
| **Post tax Cash flow** | | **-69,853** | **8,882** | **7,398** | **9,801** | **12,881** | **16,313** | **17,398** | **17,039** | **16,685** | **16,336** | **15,993** | **15,654** |
| Disc. cash flows | **6.75%** | -69,853 | 8,321 | 6,492 | 8,057 | 9,919 | 11,768 | 11,757 | 10,786 | 9,894 | 9,075 | 8,322 | |
| Disc. cash flows (Cum for 6.75%) | | -69,853 | - 61,532 | - 55,040 | - 46,983 | - 37,064 | - 25,296 | - 13,540 | - 2,754 | 7,140 | 16,215 | **24,537** | |
| Disc. cash flows | **9.5%** | -69,853 | 8,112 | 6,170 | 7,465 | 8,960 | 10,362 | 10,093 | 9,027 | 8,072 | 7,218 | 6,453 | |
| Disc. cash flows (Cum for 9.5%) | | -69,853 | -61,741 | -55,571 | -48,106 | -39,146 | -28,784 | -18,691 | -9,665 | -1,592 | 5,626 | **12,079.2** | |
| Disc. cash flows | **13.0%** | -69,853 | 7,861 | 5,794 | 6,792 | 7,900 | 8,854 | 8,357 | 7,242 | 6,276 | 5,438 | 4,711 | |
| Disc. cash flows (Cum for 13%) | | -69,853 | - 61,993 | - 56,199 | - 49,406 | - 41,506 | - 32,652 | - 24,295 | - 17,053 | - 10,777 | - 5,339 | **- 628** | |
| | | | | | | | | | | | | | |
| **CORPORATE TAX ON EBITDA** | **20%** | 2,313.60 | 5,245.80 | 5,245.80 | 5,245.80 | 5,245.80 | 5,245.80 | 5,245.80 | 5,245.80 | 5,245.80 | 5,245.80 | 5,245.80 | 5,245.80 |
| **Post tax Cash flow** | | **-69,275** | **10,194** | **8,710** | **11,112** | **14,193** | **17,624** | **18,709** | **18,350** | **17,996** | **17,647** | **17,304** | **16,966** |
| Disc. cash flows | **6.75%** | -69,275 | 9,549 | 7,643 | 9,135 | 10,929 | 12,714 | 12,643 | 11,616 | 10,672 | 9,803 | 9,005 | |
| Disc. cash flows (Cum for 6.75%) | | -69,275 | -59,725 | -52,082 | -42,948 | -32,018 | -19,305 | -6,662 | 4,955 | 15,626 | 25,430 | **34,434.3** | |
| Disc. cash flows | **9.5%** | -69,275 | 9,309 | 7,264 | 8,464 | 9,872 | 11,195 | 10,854 | 9,722 | 8,707 | 7,798 | 6,982 | |
| Disc. cash flows (Cum for 9.5%) | | -69,275 | -59,965 | -52,701 | -44,238 | -34,366 | -23,170 | -12,317 | -2,595 | 6,112 | 13,909 | **20,891.9** | |
| Disc. cash flows | **13.0%** | -69,275 | 9,021 | 6,821 | 7,701 | 8,705 | 9,566 | 8,986 | 7,800 | 6,769 | 5,875 | 5,098 | |
| Disc. cash flows (Cum for 13%) | | -69,275 | -60,254 | -53,433 | -45,731 | -37,027 | -27,461 | -18,474 | -10,675 | -3,905 | 1,969 | **7,066.9** | |
| | | | | | | | | | | | | | |
| **CORPORATE TAX ON EBITDA** | **30%** | 3,470.40 | 7,868.70 | 7,868.70 | 7,868.70 | 7,868.70 | 7,868.70 | 7,868.70 | 7,868.70 | 7,868.70 | 7,868.70 | 7,868.70 | 7,868.70 |
| **Post tax Cash flow** | | **-70,431** | **7,571** | **6,087** | **8,489** | **11,570** | **15,001** | **16,086** | **15,727** | **15,373** | **15,025** | **14,681** | **14,343** |
| Disc. cash flows | **6.75%** | -70,431 | 7,092 | 5,341 | 6,979 | 8,909 | 10,822 | 10,871 | 9,956 | 9,116 | 8,346 | 7,640 | |
| Disc. cash flows (Cum for 6.75%) | | -70,431 | -63,339 | -57,998 | -51,019 | -42,110 | -31,288 | -20,418 | -10,462 | -1,346 | 7,001 | **14,640.5** | |
| Disc. cash flows | **9.5%** | -70,431 | 6,914 | 5,076 | 6,466 | 8,048 | 9,529 | 9,332 | 8,332 | 7,438 | 6,639 | 5,924 | |
| Disc. cash flows (Cum for 9.5%) | | -70,431 | -63,517 | -58,441 | -51,975 | -43,927 | -34,398 | -25,066 | -16,734 | -9,296 | -2,658 | **3,266.5** | |
| Disc. cash flows | **13.0%** | -70,431 | 6,700 | 4,767 | 5,884 | 7,096 | 8,142 | 7,727 | 6,685 | 5,783 | 5,001 | 4,325 | |
| Disc. cash flows (Cum for 13%) | | -70,431 | -63,731 | -58,965 | -53,081 | -45,985 | -37,843 | -30,116 | -23,431 | -17,649 | -12,647 | **-8,322.4** | |