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Svante Claesson

Arbetsrapport 35 1998

SVERIGES LANTBRUKSUNIVERSITET
Institutionen för skoglig resurshushållning
och geomatik
S-901 83 UMEÅ
Tfn: 090-786 58 25 Fax: 090-14 19 15, 77 81 16

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Examensarbete i skogsuppskattning och skogsindelning
Handledare: Sören Holm
Tomas Thuresson

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35 Claesson, S. Thinning response functions for single trees of Common oak (Quercus Robur L.) Examensarbete. ISRN SLU-SRG-AR--35--SE.

Thinning response functions for single trees of Common oak (*Quercus Robur* L.).

Abstract

Functions for estimation of the total thinning response, on the basal area growth for single trees, of Common oak (*Quercus robur* L.) are presented. The total thinning response is defined as the ratio between basal area growth of a tree affected by thinnings and the hypothetical basal area growth of that same tree unaffected by thinnings. The thinning response functions can isolate the effect, on the basal area growth, of all historical thinnings in a stand, given knowledge of the intensity, time and method of every thinning performed. The functions are estimated by data from southern Sweden.

Key words: *Forest management planning, forest yield research, thinning response, Common oak, Quercus robur.*

Table of content

Introduction

During the 1980's The Forest Management Planning Package, FMPP (Jonsson et al. 1993), was developed at the Swedish University of Agricultural Science. In FMPP only the species Norway spruce (*Picea abies* (L.) Karst.), Scots pine (*Pinus sylvestris* L.) and Birch (*Betula* sp) are given special consideration, for two general reasons. These tree species are the most commercial important in Sweden, and they are relatively well studied. Other species such as Common oak (*Quercus robur* L.) are dealt with in a not very satisfactory way by the planning package.
In FMPP tree growth is predicted for individual trees. For this purpose basal area growth functions for individual trees are used (Söderberg 1986). These functions estimate the tree growth for all of the most common Swedish tree species, including Common oak (*Quercus robur* L.). To simulate the effect on tree growth of active silvicultural thinning, a special thinning response function is used. This function estimates the ratio between the diameter growth of a tree in a thinned stand and the diameter growth of a tree in an unthinned stand (Jonsson 1995). The total basal area increment, in the case of thinning, is obtained as the product of the two functions. The thinning response functions have been developed by B. Jonsson (1995) for single trees of Norway spruce (*Picea abies* (L.) Karst.) and Scots pine (*Pinus sylvestris* L.).

The existing thinning response functions are in the present version of the FMPP applied to all tree species. However, the true thinning response is likely to be different for Common oak than for conifers. The goal of this study is to construct a thinning response function for single trees of Common oak (*Quercus robur* L.) that could be implemented in the FMPP.

Material and methods

Reference material

The study is based on a data material from 26 experimental plots of oak in southern Sweden. The plots have earlier been described by Carbonnier (1975). As many as 10 of the plots are situated on the island of Visingsö, the rest of the plots are well spread in the south of Sweden. Three of the plots are located in the county of Östergötland, two in the county of Blekinge, 10 in the county of Skåne and the last one in the county of Halland. The corresponding stands are all artificially regenerated, seeded or planted. The regeneration method varies among the stands. Some of them have been planted with a spacing of 3x5 meters, while others have been seeded in rows. In some cases other species have been planted or naturally regenerated in between the seeded or planted Oaks. The oldest plots were established in 1840, and have been measured from 1898, while the youngest plot was seeded in 1940. The Oak species are in all cases Common oak (*Quercus robur* L.). There are also other species present on the plots, such as Norway spruce (*Picea abies* (L.) Karst.), Silver fur (*Abies alba* Mill.), Beech (*Fagus silvatica* L.), Larch (*Larix sp*), Ash (*Fraxinus excelsior* L.) and Birch (*Betula sp*). These species are in most cases present in the stand as undergrowth. The experimental plots are single plots on which measurements of the stands basal area and height have been made in intervals of 3-7 years. The stand basal area has been determined by cross calipering all trees over 4.5-6.5 cm at breast height. The lower limit in breast height diameter was allowed to vary depending on the stand development at the time of revision.

Table 1. *Some characteristics of the 26 experimental plots used in the study.*

Plot number	Area (ha)	H100 Oak (m)	Number of 5 year periods included in the study	Plot age at the beginning of the first period	Plot age at the beginning of the last period	Number of trees included in the study	Plot number
262	0.264	24.4	14	53	118	31	262
480	0.25	22	13	86	146	31	480
481	0.25	21.7	12	86	141	28	481
482	0.25	21.9	13	86	146	25	482
483	0.25	21.9	13	86	146	29	483
486	0.237	23.8	13	66	126	27	486
487	0.34	23.5	8	66	126	30	487
488	0.3	23.5	12	63	123	30	488
526	0.5	17.3	12	58	113	30	526
527	0.3	19	12	66	121	30	527
578	0.48	18.6	12	69	124	30	578
619	0.407	26.6	11	59	109	30	619
804	0.5	24.4	6	46	71	24	804
807	0.5	25.1	4	38	53	30	807
808	0.5	24.6	4	37	52	28	808
823	0.39	21.8	4	41	56	29	823
824	0.3	24	4	41	56	28	824
829	0.25	26.4	7	32	62	28	829
832	0.4	25.3	7	31	61	30	832
845	0.45	23.9	6	42	67	28	845
846	0.25	22.5	4	48	63	30	846
861	0.3	27.8	4	39	54	28	861
862	0.4	29	7	34	64	30	862
863	0.385	28.6	7	35	65	30	863
864	0.45	27.4	7	35	65	32	864
T60	0.255	21.4	6	41	66	29	T60

Total number of trees	755
Total number of periods	222
Total number of thinning respons observations	6192

For this study a number of trees have been selected on each experimental plot. Mainly trees present at the last measurement have been chosen, systematically according to the tree register. The intention was to select 30 trees as spatially even over the plot as possible. The selected trees have been followed from the first revision to the last. A total of 755 trees have been used, see Table 1. For every tree and revision the diameter of that tree has been recorded. This implies that one observed thinning response is obtained for every tree at every revision.

In the cases when the period between two revisions deviate from 5 years, new diameters for every tree and basal area for the stand have been calculated by means of interpolation.

The thinnings made in the stands can be defined as crown thinning and the time between thinnings varied between 3 and 15 years. To separate the effect of thinnings in the undergrowth from thinnings among the main crop trees, the undergrowth has, at each revision, been defined as those tree species and strata having a mean arithmetic height less than half the mean arithmetic height of the main crop trees. The main crop is in most cases Oak, but in some cases it consists of a mixture of species. The basal area of Oaks is on an average 81 % of the total basal area in the stands.

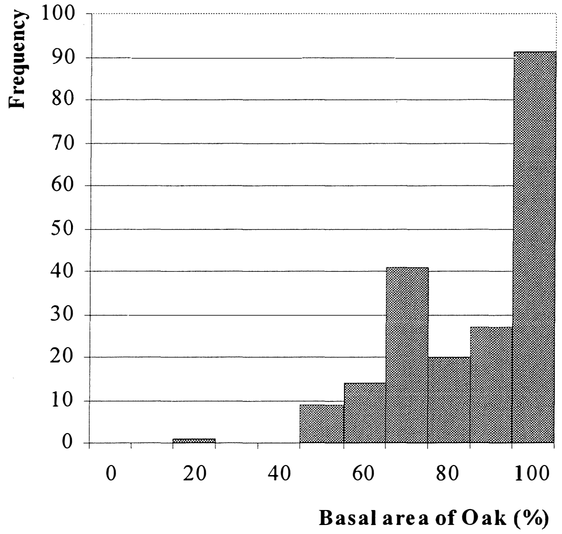

Fig 1. *Frequency of the content of Oak at every revision, expressed as basal area of Oak as percentage of the total basal area in the stand.*

The thinnings have been described by thinning intensity and diameter ratio. Thinning intensity is defined as the ratio, in percentage, of the basal area removed to the basal area in the stand before thinning. The diameter ratio is defined as the ratio between mean arithmetic diameter of stems harvested and mean arithmetic diameter in the stand before thinning. Fig 2 shows the variation in thinning intensity and diameter ratio among thinnings in the study.

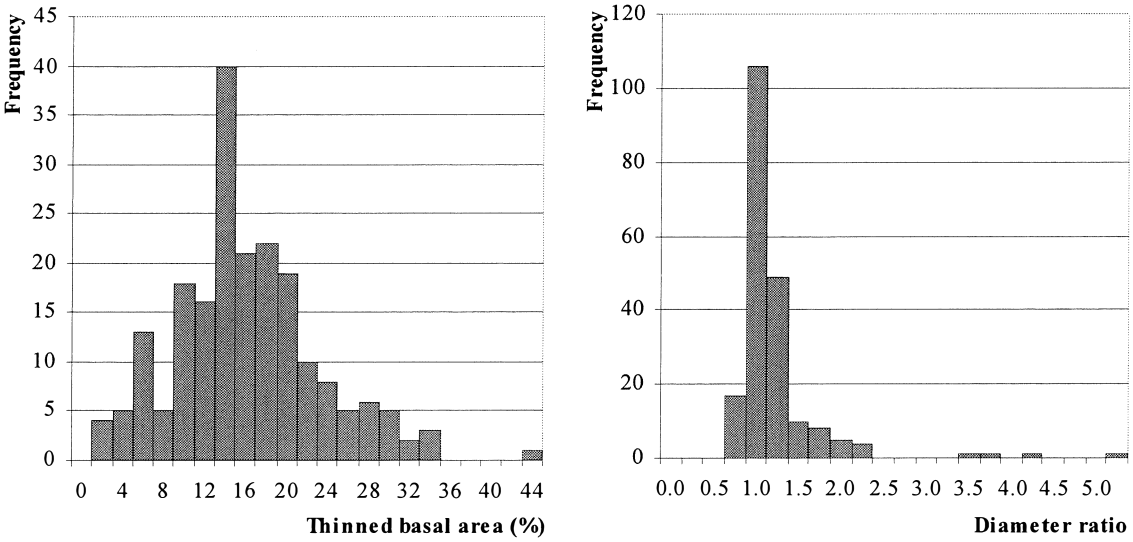

Fig 2. *Frequency of thinning intensity (left) and diameter ratio (right) for all thinnings included in the study. Thinning intensity is defined as the thinned basal area as per cent of the total basal area before thinning. Diameter ratio is defined as the ratio between the mean arithmetic diameter of the thinned stems and the mean arithmetic diameter in the stand before thinning.*

Two additional plots, for which the dominant heights were unknown at the time of the data processing, were later used for testing the estimated functions, see table 2.

Table 2. *Some characteristics of the two plots used for testing the thinning response functions.*

Plot number	Area (ha)	H100 (m)	Number of 5 year periods included in the study	Plot age at the beginning of the first period	Plot age at the beginning of the last period	Number of trees included in the study	Plot number
528	0.5	22	11	86	135	30	528
529	0.5	19.2	11	92	145	30	529

Theory behind the functions

B. Jonsson (1995) has developed a model for estimating the total thinning response, expressed in terms of the effect on diameter growth in 5-year periods. The model is briefly described below. It is developed for experimental data, with thinned parcels and unthinned control parcels both similar before the first thinning and growing under similar conditions. The model considers the time of the thinnings and the thinning intensity. The intensity of thinnings is defined as the basal area removed in percentage of the basal area in the stand before thinning. He defines G_A as the basal area removed from the stand in *active thinning* in per cent of the basal area in the stand before thinning. G_S is the percentage basal area removed from the actively thinned stand in form of *natural mortality*. The total percentage removed basal area G_T is given by:

$$(1) \qquad G_T = G_A + G_S$$

Jonsson bases his models on the assumption that the effect of active thinning and natural mortality is the same on the diameter growth of a tree. This assumption is necessary since active thinning and natural mortality are not separated in his data material. In the hypothetical stand, which has not been subject to active thinnings, the only removal of basal area is due to natural mortality. This natural mortality is denoted $G_{S'}$.

To the diameter growth Jonsson associates a growth level, which is unaffected by active thinnings and by the climate influences. This growth level is denoted lev_n, where *n* denotes the time period. The effect of the climate is given by a factor called $clim_n$. The mean annual ring width during period *n*, denoted Y_n, is described as

$$(2) \qquad Y_n = lev_n \times clim_n \times f_1(G_{T_n}) \times f_2(G_{T_{n-1}}) \times \ldots \times \theta_n$$

where θ_n is a random component, with $E(\theta_n) = 1$. The functions $f_1(\cdot), f_2(\cdot)\ldots$ are the expected thinning responses. The function describing Y_n is based on the assumption that growth factors interact in a multiplicative way. This assumption is usually made when modelling growth of trees. In the same way we obtain the growth, Z_n, for an unthinned stand with only natural thinnings and the same climate effect

$$(3) \qquad Z_n = lev'_n \times clim_n \times f_1(G_{S'_n}) \times f_2(G_{S'_{n-1}}) \times \ldots \times \theta'_n$$

By taking the ratio between the growth during period n and the growth during the period before the first active thinning, period 0, we obtain, for the thinned stand

$$(4) \qquad \frac{Y_n}{Y_0} = \frac{lev_n \times clim_n}{lev_0 \times clim_0} \times f_1(G_{T_n}) \times \ldots \times f_n(G_{T_1}) \times \frac{\theta_n}{\theta_0}$$

and for the unthinned stand

$$(5) \qquad \frac{Z_n}{Z_0} = \frac{lev'_n \times clim_n}{lev'_0 \times clim_0} \times f_1(G_{S'_n}) \times \ldots \times f_n(G_{S'_1}) \times \frac{\theta'_n}{\theta'_0}$$

In the expression (4) and (5) the *climate* ratios are equal and the growth *level* ratios can be assumed to be so too. Hence by taking the ratio between (4) and (5) Jonsson isolates the total thinning effect, as expressed by

$$(6) \qquad \frac{Y_n / Y_0}{Z_n / Z_0} = \frac{f_1(G_{T_n}) \times \ldots \times f_n(G_{T_1})}{f_1(G_{S'_n}) \times \ldots \times f_n(G_{S'_1})} \times \Phi_n$$

This is linearized by taking the natural logarithm of expression (6)

$$(7) \qquad \ln\left(\frac{Y_n / Y_0}{Z_n / Z_0}\right) = \sum_{j=1}^{n} \ln\left[f_j\left(G_{T_{n+1-j}}\right)\right] - \sum_{j=1}^{n} \ln\left[f_j\left(G_{S'_{n+1-j}}\right)\right] + \ln(\Phi_n)$$

where $\ln(\Phi_n)$ is a random component, with $\mathrm{E}(\ln(\Phi_n)) = 0$. Expression (7) is subject to linear regression analysis, where different thinning characteristics is tried for the deterministic part.

When forecasting, the basal area growth (Z_n) unaffected by active thinnings, is predicted by using growth functions, e.g. by those of Söderberg (1986). G_A is defined by the thinning programme desired to simulate. G_S and $G_{S'}$ are not known, and have to be estimated using mortality functions. The prediction of Y_n is then given by the expression.

$$(8) \qquad \hat{Y}_n = \hat{Z}_n \times \frac{\hat{f}_1(G_{T_n}) \times \ldots \times \hat{f}_n(G_{T_1})}{\hat{f}_1(G_{S'_n}) \times \ldots \times \hat{f}_n(G_{S'_1})} \qquad (n > 0)$$

Application of the theory to the material

In this paper the thinning response of a tree is defined as the ratio between the actual growth in basal area of a tree affected by thinnings, and the hypothetical growth in basal area of that same tree if unaffected by thinnings. The total thinning response is the accumulated result of all previous active and natural thinnings. This definition is identical to the definition used by Jonsson (1995), except that he uses the ratio between diameter growths.

The data material available does not include unthinned control plots, which means that there are no values of growth, Z_n, from an associated unthinned stand. Therefore the values of Z_n have been estimated through growth functions developed by Söderberg (1986). This function predicts the growth of basal area during a 5-year period for single trees of Oak. Another limitation of the data material used is that the value of Y_0, the basal area growth during the period before the first thinning is not known.

Still assuming the model (2), the logarithm of the ratio Y_n / Z_n equals

$$\ln\left(\frac{Y_n}{Z_n}\right)=\sum_{j=1}^{n}\ln\left[f_j\left(G_{T_{n+1-j}}\right)\right]-\sum_{j=1}^{n}\ln\left[f_j\left(G_{S'_{n+1-j}}\right)\right]+\ln\left(lev_n / lev'_n\right)+\ln(c\,lim_{\mathrm{n}})+\ln\left(\Phi_n\right) \tag{9}$$

This means that the effect of *climate* and the ratio of lev_n and lev'_n do not cancel. The factor of *climate* is considered here as a random component, while the *level* ratio might express some systematic difference between the data and the function.

Two basically different types of expressions have been used for expressing the function $f_j(\cdot)$. These two expressions are the same as those used by Jonsson (1995). The first expression does not consider the diameter ratio of the thinning, but the second one does.

Regression model I is defined by

$$\ln Q_n=\sum_{j=1}^{n}\alpha_j \times G_{R_{n+1-j}} \tag{10}$$

where $\ln Q_n$ is the deterministic thinning effect in expression (9). $G_{R_{n+1-j}}$ is defined by

$$G_{R_{n+1-j}}=G_{A_{n+1-j}}+G_{S_{n+1-j}}-G_{S'_{n+1-j}}$$

$G_{A_{n+1-j}}$ is the percentage actively thinned basal area of the thinned stand, and $G_{S_{n+1-j}}$ is the natural mortality of the thinned stand. $G_{S'_{n+1-j}}$ is the natural mortality of the hypothetical unthinned stand, and have been estimated using mortality functions by G. Bengtsson, see Hägglund (1981).

Regression model II is defined by

$$\ln Q_n=\sum_{j=1}^{n}\alpha_{n+1-j}\times\sqrt{dq_{n+1-j}}\times G_{R_{n+1-j}} \tag{11}$$

Where *dq* denotes the ratio between mean arithmetic diameter of the thinned stems and the mean arithmetic diameter in the stand before thinning.

Model I has also been used with a separation between thinnings in the understory and among the main crop trees. This results in model III, which can be written as

$$ln\,Q_n=\sum_{j=1}^{n}\alpha_j \times G_{MR_{n+1-j}}+\sum_{j=1}^{n}\beta_j \times G_{UR_{n+1-j}} \tag{12}$$

where G_{MR} is the percentage of basal area removed from the main crop trees, and G_{UR} is the percentage of basal area removed from the understory.

Results

Thinning response functions

Thinning response functions have been estimated using linear regression analyses according to the models presented above. The material consists of trees that have been measured up to fourteen 5-year periods for the oldest stands. It would have been possible to include more historical thinnings, but the thinning response showed to be significant for only 7 historical thinnings. According to the thinning response functions, the response of one thinning lasts for at least 35 years.

Three thinning response models are presented in Table 3. A systematic difference, as suggested by expression (9), was found to be a second-degree function of *age*. Hence, the *constant* and the variables *age* and age^2 have been included when estimating the functions to calibrate the growth functions. These variables should not be used when applying the thinning response functions.

Model II considers the diameter ratio of the thinnings, but model I and model III does not. Model III separates thinnings in the undergrowth from thinnings among main crop trees. The effect, on the main crop trees, of thinnings in the undergrowth was found to be significant only in the most recent thinning. This might lead to the conclusion that the thinning effect on the main crop trees of thinnings in the undergrowth only last for about 5 to 10 years. The function in Table 4 shows that the effect of a 1% thinning among the main crop trees gives a thinning effect of 1.1% on the basal area increment of the remaining Oaks, while a 1 % thinning in the undergrowth only gives a 0.053% thinning effect. Undergrowth is in this case defined as tree species and strata that have half the mean arithmetic height of the mean arithmetic height of the main crop trees.

To simplify, the thinning intensities are below expressed as G_1, G_2, ... where G_1 is the last thinning intensity before the growth period at hand, instead of the chronological indexation used in the model section.

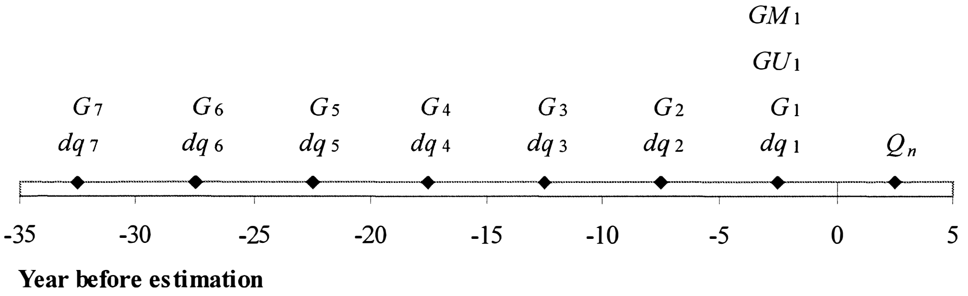

Fig 3. *A time-scale showing the period in which the variables, from the thinning response functions are occurring. The variables are explained in Table 3.*

Table 3. *Thinning response functions for single trees of Common oak (Quercus robur).*

The dependent variable is the natural logarithm of the total thinning response, $\ln(Y_n / Z_n)$. The independent variables consist of the thinning grade, expressed as the removed basal area in percent of the total basal area in the stand before thinning, G_p. The variable dq_p is the diameter ratio between the mean arithmetic diameter of the thinned stems and the mean arithmetic diameter in the stand before thinning. In model III GU_1 denotes the percent thinned basal area in the undergrowth, and GM_1 denotes the percent thinned basal area among the main crop trees, during the last 5-year period before the year when the thinning response is estimated.

For each function the standard error of the estimate (SD), R^2 and S_{PRESS}/S_{REGRES} is given. R^2 is calculated as

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

where y_i is the *i*th observation.

A ratio of S_{PRESS}/S_{REGRES} lower than 1.05 indicates that the function is not over-fitted. S_{REGRES} is the standard deviation about the regression function, and S_{PRESS} is *PRESS/n*, where *PRESS* is a residual sum of squares produced by means of cross-validation (Weisberg 1985).

	Model I			Model II			Model III	
Variable	α	Sig. t	Variable	α	Sig. t	Variable	α	Sig. t
(Constant)	-1.4168	0.000	*(Constant)*	-1.3897	0.000	*(Constant)*	-1.4295	0.000
Age	0.03269	0.000	*Age*	0.03401	0.000	*Age*	0.03227	0.000
Age^2	-0.0001788	0.000	Age^2	-0.0001875	0.000	Age^2	-0.0001720	0.000
G_1	0.01115	0.000	$\sqrt{dg_1} \times G_1$	0.00818	0.000	GU_1	0.000530	0.020
G_2	0.005445	0.000	$\sqrt{dg_2} \times G_2$	0.004417	0.000	GM_1	0.01114	0.000
G_3	0.004733	0.000	$\sqrt{dg_3} \times G_3$	0.003809	0.000	G_2	0.005248	0.000
G_4	0.001694	0.023	$\sqrt{dg_4} \times G_4$	0.001582	0.045	G_3	0.004547	0.000
G_5	0.002665	0.001	$\sqrt{dg_5} \times G_5$	0.002099	0.014	G_4	0.002892	0.000
G_6	0.004020	0.000	$\sqrt{dg_6} \times G_6$	0.003719	0.000	G_5	0.003108	0.000
G_7	0.005691	0.000	$\sqrt{dg_7} \times G_7$	0.005097	0.000	G_6	0.004677	0.000
						G_7	0.005252	0.000
SD		0.4622			0.4662			0.4632
R^2		0.2310			0.2176			0.2278
S_{PRESS}/S_{REGRES}		1.0005			1.0006			1.0006

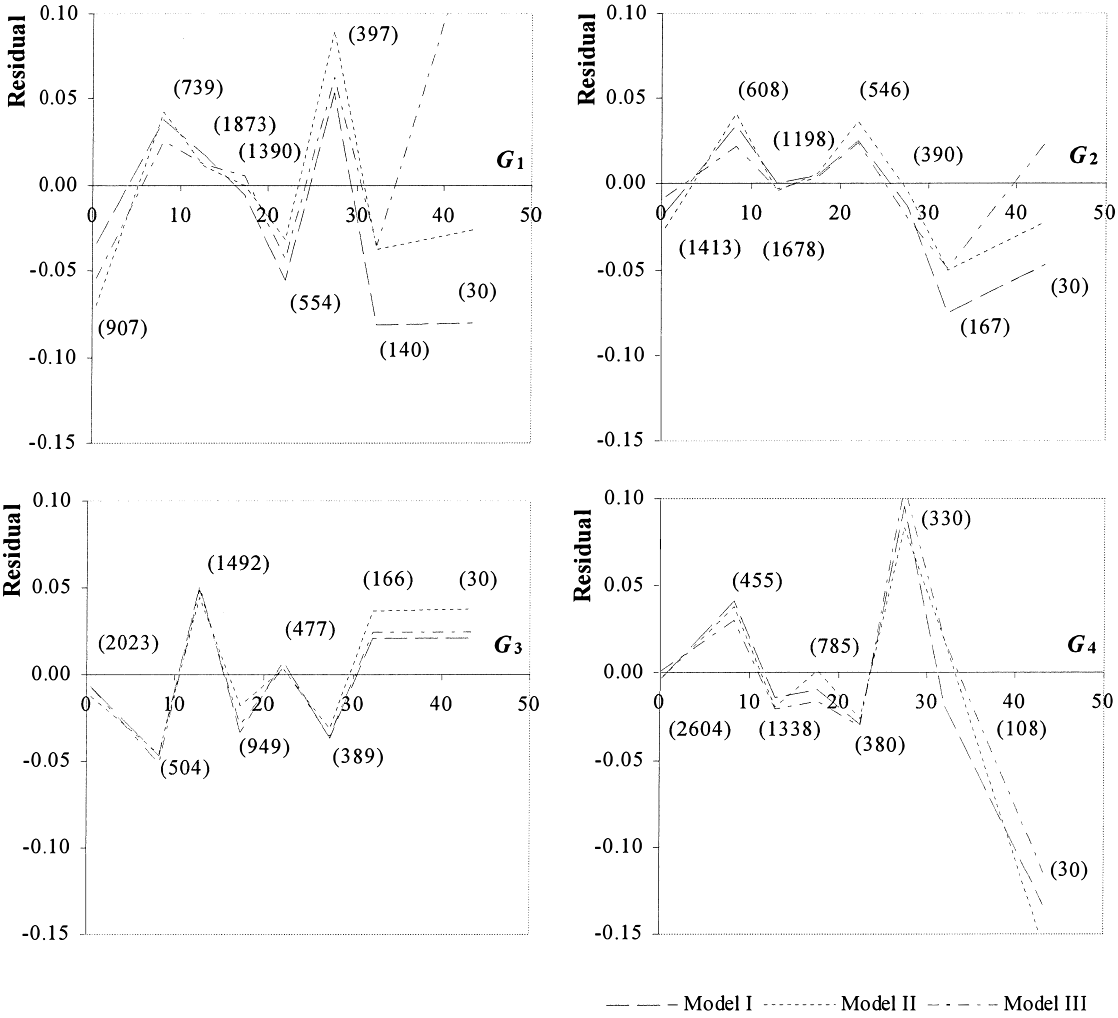

Fig 4. *Residuals of the three functions plotted against the removed basal area in percentage of the basal area in the stand before thinning. G_1, G_2 etc denotes the basal area removed in percentage of the basal area in the stand before thinning, in period 1, 2 etc before the period in which the estimation is done. Numbers inside brackets is the number of observations for each mean value of residuals.*

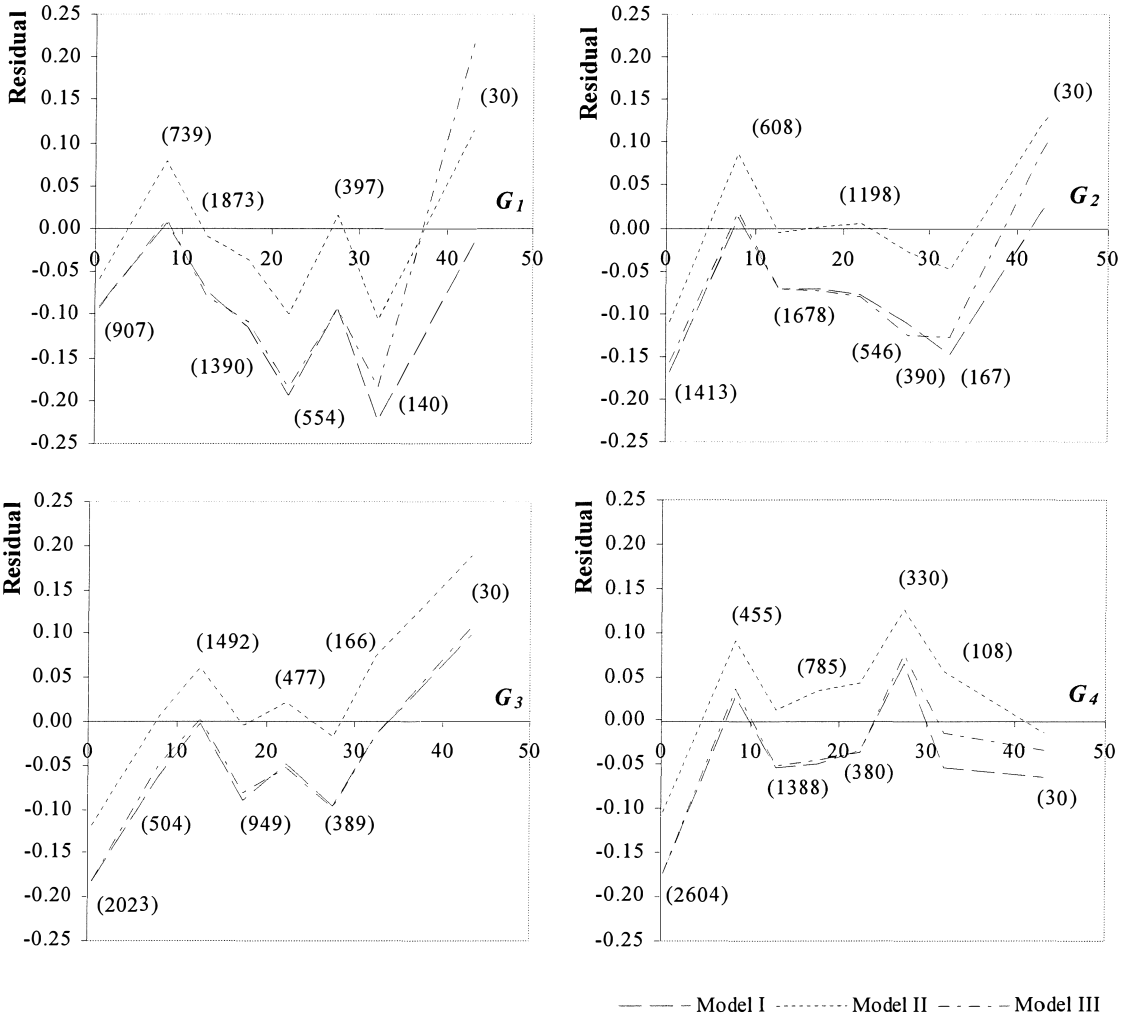

Fig 5. *Residuals of the three functions, calculated without the constant and the variables of age and age^2, plotted against the basal area removed in percentage of the basal area in the stand before thinning. G_1, G_2 etc denotes the removed basal area in percentage of the basal area in the stand before thinning, in period 1, 2 etc before the period in which the estimation is done. Numbers inside brackets is the number of observations for each mean value of residuals.*

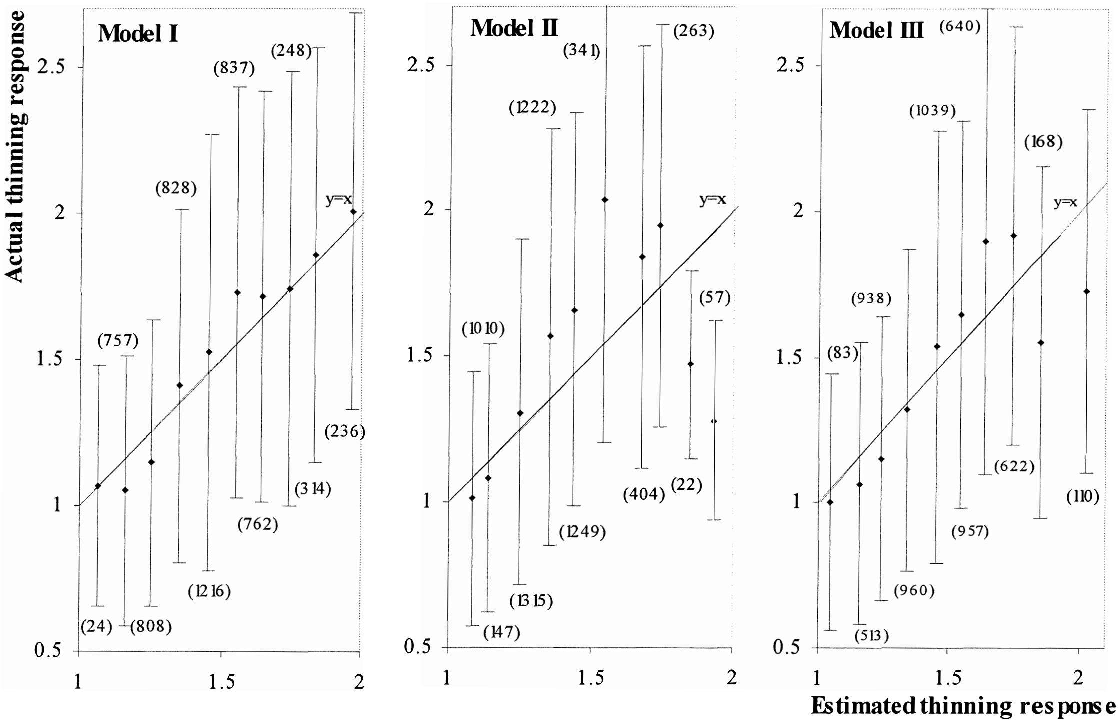

Fig 6. *Comparison between the observed mean (dots) and estimated total thinning response. The estimated thinning response is calculated with the thinning response functions presented in Table 3, without the constant and variables of age and age^2. Vertical lines represent the standard deviation in the data material (residuals), and numbers in brackets represent the number of thinning response observations for each mean value of observed thinning response.*

Testing the functions

The resulting three functions were tested against the real growth of two plots. Both plots are situated on the island of Visingsö, and have a similar history. For each 5-year period, and each plot, the growth in basal area for 30 systematically chosen trees have been estimated as

$$(13) \qquad \hat{Y}_n = \hat{Z}_n \times \hat{Q}_n$$

where $\hat{Z}_n$ is estimated using Söderberg's (1986) growth functions of basal area for single trees of Oak, and $\hat{Q}_n$ is estimated using the three different thinning response functions presented above.

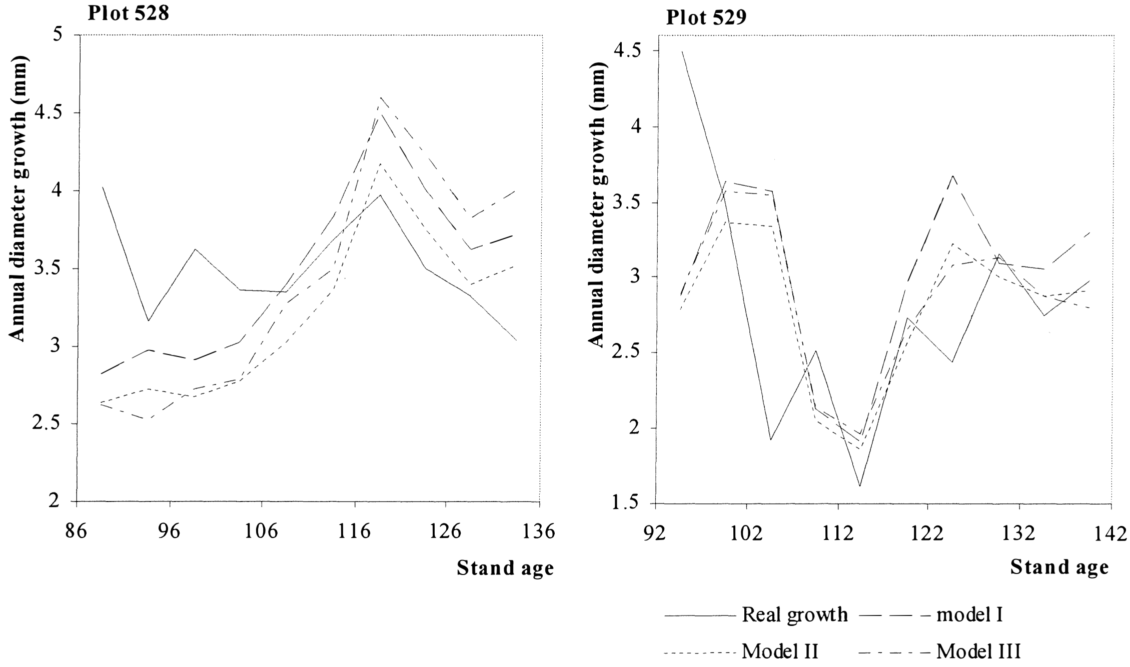

Fig 7. *Mean annual diameter growth, of the two plots, compared with the estimated annual diameter growth.*

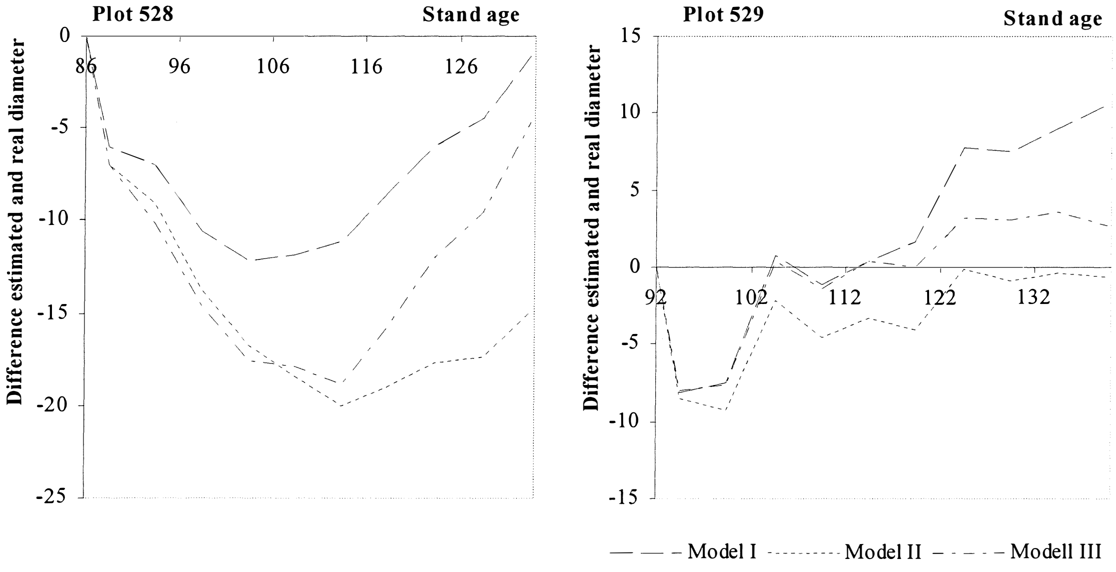

Fig 8. *Differences between estimated and real diameter development, of the two plots.*

Components of error

Since observations are hierarchically ordered in trees and plots, it could be interesting to partition the total variance into its components. By using the Mixed procedure in the SAS statistical system (SAS Institute Inc 1992), the total variance of the three presented models was divided according to

$$\sigma^2_{total} = \sigma^2_{plot} + \sigma^2_{tree} + \sigma^2_{residual}$$

Table 4 shows that the three components of variance is of the same magnitude, approximately 30 % of the total variance each, for the three models.

Table 4. *Estimated components of variance for the three regression models.*

Model	σ^2_{plot}	σ^2_{tree}	$\sigma^2_{residual}$
I	0.0882	0.1261	0.1006
II	0.1111	0.1253	0.1027
III	0.0941	0.1260	0.1016

Site quality class and thinning response.

The data material has been divided into three site index classes to see how the site index influences the total thinning response. A regression analysis according to model I was made with the grouped material. The grouped material only resulted in significant values to the estimated thinning responses for three 5-year periods. Fig 8 shows that a higher site index gave a lower response to thinnings, than a lower site index.

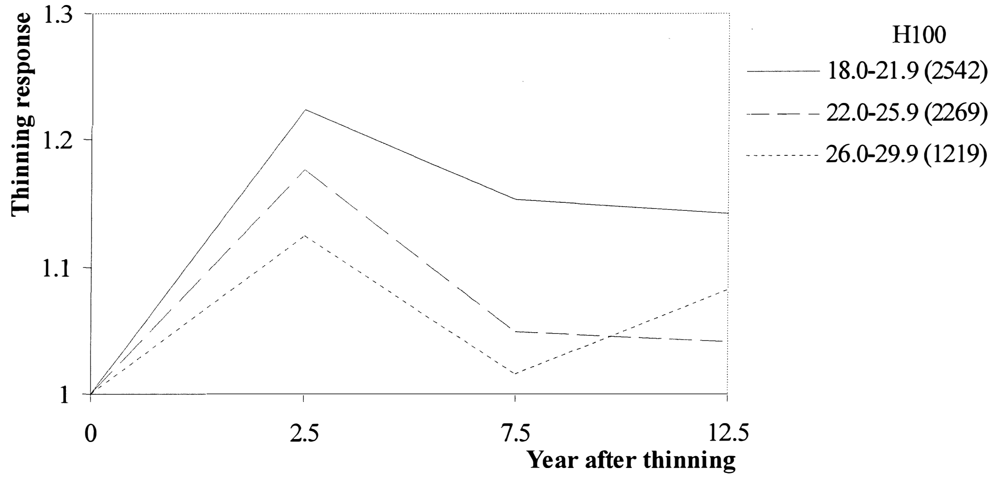

Fig 9. *The difference in thinning response between different site quality classes after one thinning where 15 % of the basal area have been removed. Numbers inside brackets is the number of thinning response observations for each site quality class.*

Visingsö

In a work by Carbonnier (1951) it is reported that the experimental plots from the island of Visingsö showed a lower growth in basal area than found in other growth studies of Common oak in Scandinavia. He meant that the reason for this was the management of these stands, mainly because the basal area of the stand had been held low. The Visingsö data was separated from the remainder of the material, to see if the thinning response differed between the two groups. Out of the total number of 6192 thinning response observations, 3411 are from the island of Visingsö. Two functions were created by means of linear regression, one with only data from the Visingsö plots and one with data from the remaining plots. An F-test was calculated to see if the difference in coefficients, between the two functions, was significant. The difference in coefficients showed to be significant at the 1 % level. The Visingsö group showed a slightly higher and faster response to thinning than the other group. With a thinning program of a 15 % basal area removal during five 5-year periods, the Visingsö function shows only a 0.3 % higher thinning response.

Discussion

Thinning response functions for single trees of Common oak (*Quercus robur* L.) have been estimated using linear regression. The dependent variable is the natural logarithm of the ratio of basal area growth of single trees in a thinned stand and the expected basal area growth according to a growth function. The developed functions are intended to be used to predict the basal area growth of single trees of Common oak, together with a function for prediction of basal area growth of single trees of Common oak in an unthinned stand, as in expression 13 (p. 15).

The thinning response functions developed by Jonsson (1995) for single trees of Norway spruce and Scots Pine are based on data material where the diameter growth of every tree is measured on bore cores. In this study the diameter growth is measured by calipering the trees in two directions at breast height. This is one cause for the greater variance in the material in this study. Two other causes for greater variance is the fact that climate effects are not accounted for and that it has not been possible to adjust each tree for its historical growth. The standard deviation for model I (p. 11) is approximately twice as high as the standard deviation of the functions developed by Jonsson.

Table 4 shows that the tree dependent and plot dependent errors are as large as the pure random error component. The tree error variance, most likely, would have been smaller if the data material had contained a reference growth for each tree, as in Jonsson (1995. p 361). The plot error variance has the same magnitude as in the study by Jonsson. Inserting plot variables into the function could possibly reduce the plot error variance. In that case, to ensure no thinning response in case that no thinning has been performed, plot variables have to be entered in a multiplicative combination with the thinning intensity. Expressions tested in this way gave no significant results.

The extent in which the understory trees in a stand of Common oak affects the growth of the main crop is discussed by Carbonnier (1951). He found that the basal area increment of the main crop trees is affected by the growth of the understory in such a way that the increment in basal area of the main crop tends to decrease when the basal area increment of the undergrowth increases. He also found that old Oak stands were more sensitive to undergrowth than young ones. Other studies have shown no effect on the Oaks basal area increment caused by undergrowth (Bornebusch 1948). Wiedemann (1942) found a 7 % decrease in basal area increment due to undergrowth of Beech (*Fagus silvatica* L.). If the undergrowth affects the growth of the main crop trees there should be a thinning effect, on the main crop trees, due to thinning in the undergrowth. Model III, presented in Table 3, confirms that the presence of an understory would affect the growth of the main crop. Table 3 also shows that the thinning response on the main crop due to thinnings in the undergrowth is only 5 % of the thinning response due to thinnings among the main crop, given the same intensity.

The thinning response functions presented in this study might lead to the conclusion that the thinning response from one thinning will last for 35 years. This is only true provided continued thinnings. All plots used to create the thinning response functions are thinned in even interval. Consequently there is no thinning response observation in the material where the thinning response is estimated from only one thinning, made 35 years ago.

Fig 8 indicates that the thinning response of Common oak seems to be lower for a high site index than for a low site index. Jonsson (Jonsson et al. 1993) found the opposite relationship between thinning response and site quality class, for thinnings made in stands of Norway spruce and Scots pine. The data material, grouped into site quality classes, contains too few observations in each group to give significant functions, with seven historical thinnings.

The diameter ratio is an important variable since the material used includes thinnings in the undergrowth. As shown earlier, thinnings in the undergrowth gives a lower thinning response than thinnings among the main crop trees. The diameter ratio effect shows a lower thinning response for thinnings from below for a given thinning intensity.

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