Spatial continuity in tree diameter distribution

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Master thesis in Forest Biometry
Supervisor: Sören Holm, Swedish University of Agricultural Sciences

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Abstract

Investigation for spatial continuity in tree diameter distribution is attempted. Parametric and non-parametric approaches are employed to see whether tree diameter distribution acts as regionalized variable or not. Tree diameter at breast height (dbh) with size five cms or greater is recorded in circular plot along with the relative geo-referenced co-ordinates of the plot center at compartment level. For the parametric approach, truncated two parameter Weibull distribution is found best fit for plot data and that an overlook of truncated data may result in wrong inference. From the truncated Weibull distribution, the maximum likelihood scale and shape parameter estimates of each plot data is obtained. Consequently, both parameter estimates, as attributes of tree dbh distribution, are subjected to spatial variation study using omnidirectional semivariogram and associated models at compartment level. Computation of cumulative distribution function of dbh at selected cutoff (cdf) in the plots is followed by investigation for spatial continuity in the non-parametric case. The spatial continuity study of the number of stems per hectare greater or equal to the selected cutoff is also considered.

The variograms and cross validation study of the Weibull parameter estimates, cdf and number of stems as attributes of tree dbh distribution seem to be very indicative to suggest that tree diameter distribution exhibits spatial continuity in the same fashion to regionalized variable.

Key words: Spatial continuity; Geostatistics; Regionalized variable; Truncated distribution function; Variogram; Kriging; Cross validation.

1. **Introduction**

In natural resources, including forestry, estimation and mapping remain to be an integral part of management, planning and research. Estimation and mapping require data acquired through a survey of the resources. For reasons of cost, time, and practicality total survey of resources is rarely possible. Accordingly, an appropriate sampling design is necessary. However, there is a widely reported precaution that classical statistical methods have overlooked the spatial characteristics of properties in sampling and subsequent analysis(Matheron, 1963; Cressie,1993; Isaaks and Srivastava, 1989; Biondi et al., 1994; Rossi, 1991; Köhl and Gertner, 1997).

Matern (1960) reported problems related to spatial variation in sampling, especially in forest survey and designs of field experiments. He warrants to have a good knowledge of spatial variation in regions where sample survey or field experiments are to be carried out. Analysis of forest structure and dynamics that include information on spatial variation are bound to give a more accurate description of reality (Biondi et al., 1994). Holmgren and Thuresson (1996) suggested allocation of treatment units on the grounds of spatially continuos description of the forest which they termed as "dynamic treatment units". They attempted to justify that dynamic treatment units improve the economic output of forest management. Hof et al. (1996) investigated the implications of the spatially autocorrelated forest yield in relation to harvesting cost optimization. The study lead them to general conclusion that consideration of spatial dependency in forest ecosystem management may be quite important in managing risk and uncertainty. Biondi et al.(1994) noted the importance of spatial knowledge to define homogeneous unit areas of forest ecosystems with respect to a given variable or set of variables which in turn lies a ground to design optimal sampling schemes, apply efficient silvicultural treatments and reduce management costs.

It is, therefore, of paramount importance to study the spatial variation of forest attributes and to take account this spatial features in all subsequent silvicultural and management decisions.

The attempt of this study is to investigate spatial continuity of tree diameter distribution and thereby the possibility to estimate the diameter distribution on the basis of spatial continuity functions. In general, there are two approaches that may exploit the spatial continuity models so that kriging method can be used to estimate the distribution of an assumed property (Isaaks and Srivastava, 1989). These are parametric and non parameteric approaches. The parametric method requires distribution function (e.g the family of two parameter weibull distribution) that is assumed to fully describe the distribution of the tree diameters (property under study). Then the spatial continuity function will be constructed on the basis of the parameters estimated from the assumed distribution function of the plots. Consequently, it makes possible to use kriging method to estimate parameters of the expected diameter distribution function.

The non-parametric approach requires no distribution function assumption. Rather, it is based on spatial continuity models of diameter size proportion below or above certain thresholds or cutoffs. To describe the diameter distribution in this way require to construct variogram models at several cutoffs of the diameter distribution. It is similarly possible to speak of spatial continuity models for proportions between certain cutoffs or class interval. The former method in this approach leads to cumulative distribution and the latter to frequency distribution estimation where one can be obtained from the other.

This work is, therefore, limited to:

1) investigate a distribution function that well describes the diameter distribution on plot level of the forest under consideration,
2) investigate spatial continuity of the parameters estimated from the selected distribution function in(1),
3) examine spatial continuity of the proportion of diameter size and number of stems per hectare at selected cutoff, and
4) carry out cross validation study of the fitted models.

2. **Geostatistical theory**

According to Matheron(1963) Geostatistics is "...concerned with the study of the distribution in space of useful values for mining engineers and geologists ...". This signifies Geostatistics is an applied statistics that originates and develops from mining industry. Through time, Geostatistics has proven to be applicable in many fields and it is now neither limited to geology nor becomes the only concern of mining engineers. Isaaks and Srivastava (1989) defines Geostatistics as a method for describing the spatial continuity that is an essential feature of many natural phenomena. In general, Geostatistics is a package of statistical tool that comes to be a widely accepted for understanding, describing and ultimately estimating values of observations in space. Thus, the concern of this applied statistics is to take account the space characteristic (spatial variation) of the observations which was thought to be overlooked in classical statistics. Such observations viewed in the totality of their set-up (location) defined by Matheron (1963) as regionalized variables, simply to stress the spatial aspect of the phenomena.

In its simplicity form, Geostatistical tools assume that neighbouring values (samples) are not independent to each other. Rather, in general terms, closer samples posses similarity and this similarity diminishes with distance to a level called discontinuity or nugget effect in Geostatistical terms. The tools of Geostatistics describe and model this autocorrelation relation and then uses the model to estimate unvisted values closer to the samples.

Matheron (1963) summarized the general properties of the regionalized variable as following:

a) It is localized moreover defined by its geometrical support(holder),
b) Its continuity as spatial variable ranges from steady continuity to discontinuity (nugget effect), and
c) There may exist preferential direction where the values do not vary significantly (anisotropy).

The common functions of Geostatistics found worthy in describing the spatial continuity of regionalized variables include variogram (semi-variogram), correlogram and covariogram. Variogram is the most common tool. The use of variogram is related to two stationary conditions which constitute the intrinsic hypothesis of regionalized variable theory (McBratney &Webster, 1986; Oliver and Webster, 1990). These are: a constant local mean and stationary variance of the differences between values in places separated by a given distance and direction. To properly describe and model the spatial variation of the regionalized variables, it is therefore required that the intrinsic hypothesis holds true (McBratney & Webster, 1986; Köhl & Gertner, 1997; Oliver & Webster, 1990).

Classical variogram estimate is computed (Matheron,1963) as:

$$\hat{\gamma}\ (\mathrm{h}) = \frac{1}{2N(h)}\sum_{i}^{N(h)}(Z(x_i) - Z(x_i + h))^2$$

where N(h) is the number of pairs of values separated by h, lag , and Z(.) is the value of interest at a given location. The estimates are expected to yield an increasing function with distance, h, since on average the further the two samples are the more they look different and consequently the variance of their difference increases. The sample variogram estimates are plotted against the distance h to examine directional and scale of dependency of the regionalized property. Moreover, this plotted function also serves as a basis to choose a model for the variogram.

To avoid problems related to parameter estimation, a commonly known as positive definite (authorized) functions are used to model the variogram estimates. The basic variogram models can be conventionally divided into those that reach a plateau and those that do not. For the formulas of variogram models may be consulted to McBratney & Webster,1986; Isaaks and Srivastava, 1989 and Cressie, 1993.

The Spherical, Exponential and Gaussian models are often referred to as transitional models since they reach a plateau. The linear model is an example of the models that do not reach a plateau and referred to as non transitional model. The distance at which the variogram model reaches a plateau is called the range. The corresponding variogram to the range is called the sill. The Exponential and Gaussian models reach the sill asymptotically. Consequently, their range is commonly defined as the distance at which the variogram value is 95% of the sill (Isaaks and Srivastava, 1989).

The method of moments, maximum likelihood, and restricted maximum likelihood are common methods for estimating parameters of variogram models (Curriero and Lele, 1999). Curriero and Lele also studied and recommended use of composite likelihood method. Even though, it is not without limitations, the moment method is the most popular for model fitting. Fitting variogram models using least squares approach including ordinary non-linear and weighted least squares are the common moment methods. Zimmerman and Zimmerman (1991) compared different estimation methods and noted the importance of ordinary non-linear least squares (NLS) and weighted non-linear least squares. Cressie (1985) suggested weighted non-linear least squares (WLS) parameter estimator:

$$\sum_{j=1}^{K} \mathrm{N(h(j))} \left\{ \frac{\gamma(h(j))}{\gamma(h(j);\phi)} - 1 \right\}^2$$

where N(h(j)) pairs in lag j, K is number of lags, $\gamma(h(j))$ value of empirical variogram at lag j and $\gamma(h(j);\phi)$ is the known variogram model with the unknown , ϕ , vector of parameters.

Variogram summarizes the general form of variation, its magnitude and spatial scale. It is useful to compare variation of properties within a region and most importantly for optimal interpolation or kriging (Oliver and Webster, 1990).

Kriging is best linear unbiased estimator that allows to estimate value of unvisited location using weighted linear combination of the local sample values with its desirable quality in minimizing the error variance (Isaaks and Srivastava ,1989).

In the absence of a data set aside to validate the spatial predictor, a cross validation is a common approach (Ripley,1981; Cressie, 1993; Isaaks and Srivastava,1989; Mcbratney and Webster, 1986). This cross validation refers to the deletion of each datum in turn and predicting it from the rest sample using the fitted model. For instance, delete a datum Z(s_j) and predict it with $\hat{Z}(s_j)$ from the variogram model fitted and the sample data **Z** excepting Z(s_j). This procedure is applied for all n elements of the sample data **Z** to produce the set of n predicted values, each associated with its mean squared prediction error $\sigma^2(s_j)$.

The closeness of the estimates to the true value can be assessed in various ways. Isaaks and Srivastava (1989) demonstrated some package of tools including a quantile - quantile and scatter plot of the predicted versus the observed values. Cressie (1993) has demonstrated the use of the standardized prediction residuals for the purpose as shown in the following equations:

$$\frac{1}{n}\sum_{J=1}^{n}\left\{\left(Z(s_j)-\hat{Z}(s_j)\right)/\sigma(s_j)\right\} \qquad (1)$$

$$\left[\ \frac{1}{n}\sum_{J=1}^{n}\left\{\left(Z(s_j)-\hat{Z}(s_j)\right)/\sigma(s_j)\right\}^2\ \right]^{1/2} \qquad (2)$$

steam and leaf plot of $\left\{(Z(s_j)-\hat{Z}(s_j))/\sigma(s_j): j=1,\ldots,n\right\}$. (3)

To feel a confident on the spatial prediction model and the mean squared prediction error, the mean in (1) and the root-mean-square in (2) should be approximately zero and one, respectively (Cressie, 1993). The histogram in (3) is used to detect outliers.

Isaaks and Srivastava (1989) have also remarked that the statistics of the reduced residuals (3) from a cross validation study are commonly used as indications of how well the variogram model is performing in practice. McBratney & Webster (1986) have demonstrated the use of the standardized predicted residuals which is also commonly known as reduced residuals for choosing fitted models.

3. Geostatistical works in forestry

According to Matern(1960) report, use of spatial variation concept in forest work is dated back to 1926. He noted Langsaeter (1926) used the semivariogram incidentally to express variation when dealing with systematic sampling in forest survey.

Reports of Matern(1960) also show that Matern (1947) has carried out a spatial variation study using different variables from the National Forest survey of Sweden in which, among others, the spatial variation of volume trees was presented using correlograms. Matern(1960) examined the spatial continuity of coniferous seedlings observed in plots of 0.7m radius using covariance function. The material is collected from 196 plots lied out in an experimental field established by Swedish Forest Research Institute. Oliver and Webster (1987) have shown spatial distribution of soil in Wyre Forest in England and demonstrated variogram model selections and kriging estimation. Using height growth of plantation of Dahrek (Melia azedarach Linn.) in an experimental area of about half hectare, Samara et al.(1989) dealt with variogram models and kriging estimation. Spatial variation study of soil property in secondary tropical dry forest observed in a 56m x 56m grid was studied and exhibited some spatial autocorrelation at a distance of 24m or less (Gonzalek and Zak, 1994). Gonzalek and Zak also carried out block kriging procedure to estimate unsampled locations. Höck et al. (1993) constructed linear model for the variograms of forest site index and satisfactorily used for kriging estimation. Biondi et al. (1994) have used variogram models and kriged maps to study spatial dependence of stem diameter (DBH), basal area (BA) , and 10 year periodic basal area increment(BAI) in an old growth permanent plot stand. They come to conclude that spatial dependence explained a large amount of stem size variability, a Gaussian model remarkably fits well the omnidirectional sample variograms for DBH and BA, variogram models for DBH and BA are consistent through time (with or without ingrowth records), and the range for the variograms was 30m. Biondi et al. also reported that spatial dependence of stem increment was smaller and decreased through time than that of stem size. Köhl and Gertner (1997) attempted to describe the spatial distribution of forest damage, needle/leaf losses based on average needle/leaf loss of sample plots over 4km x 4km grid. They fitted exponential variogram models and applied block kriging for estimation to demonstrate the potential of Geostatistical methods in forest inventory. Kuuluvainen et al.(1996) investigated the spatial autocorrelation of tree size (DBH, and height) in managed and primeval forest in Finland over an area with size 50m x 50m. They found clear spatial dependence in tree size up to inter tree distance of about 12 meters in managed forest and weak spatial continuity in the primeval forest. Using Spherical variogram model, Gunnarsson et al. (1998) have applied kriging to some useful forest variables including volume, volume increment, and site index in stratified forest.

The summary of the literature work indicates a growing concern to incorporate the spatial dependency of forest attributes to planning and management. However, as far as the knowledge of the author is concerned, no effort is made to study the spatial continuity of diameter distribution using the parametric or non-parametric approaches.

4. **Materials and Methods**

4.1 **Materials**

The data is collected on August 1998 from the forest owned by the County Board of Forestry in Västerbotten, Sweden. The area is located at Bäcksjön, 15 Kms north of Umeå (latitude $63^0 50'$N, longitude $20^0 30'$E). The elevation of the area is on average 100m above sea level. The main data of interest consists of tree diameter at breast height with five or more cms size and relative geo-referenced co-ordinates of the center of each plot. Six compartments are considered for the study based on the map of the forest. Main plot locations are laid out on the map of each compartment with a grid square using systematic random sampling design. Additional short interval plots were also laid out from the main plots alternatively on the N, S, E and W directions. For details on sampling design description see Appendix 1. In the forest, circular plots with radius 10 meters are established for inventory using compass and measuring tape. Inventory was carried out using the Forest Management Planning Package (Jonsson et al., 1993). The relative geo-referenced co-ordinates of each plot centre is also recorded using the map, distance and direction information. The main species in this mixed forest are Scots pine (*Pinus sylvestris*) and Norway spruce(*Picea abies*) with low presence of two broad leafed species: Birch(*Betula spp.*) and Aspen (*Populus tremula*). The general features of the data is shown in Table 1.

Table 1: General features of the data

a) Measured attributes

Comp. No.	Area (ha)	Number of plots	Ave. Trees /plot	D_{GW} Cm	G m^2/ ha	St/ ha	Tree Sps. (%) by stem Pine	Spruce	Broad sps
17	7.3	39	25.1	26.30	32.37	800	23.25	75.38	1.37
34	3.0	24	39.6	18.55	25.60	1262	64.84	29.88	5.28
39	1.9	36	31.3	25.9	25.43	996	25.31	48.68	26.01
41	1.4	16	36.7	17.65	19.70	1170	59.66	29.74	10.60
43	10.1	38	47.4	16.9	23.93	1511	37.43	44.82	17.75
45	2.9	22	37.0	14.8	15.35	1178	96.35	2.46	1.19

b) Less objective attributes

Comp.	Total age (years)	Thinning	Site quality index
17	100 : fairly even	No: last 15 years	19(spruce): with moderate variation
34	60: even	Partly: last 15 years	18(pine): almost no variation
39	100: uneven	No: last 15 years	20(spruce): heterogeneous stand, and partly swampy area
41	60: fairly even	Partly: some last five years	19(pine): almost no variation
43	70: uneven	No: last 15 years	16(spruce): moderate variation, heterogeneous stand
45	50: even	Partly: last 5 years	18(pine): moderate variation

4.2 **Methods**

4.2.1 **Parameter estimation for tree diameter distribution**

Knowledge of stand wise tree diameter distributions is central in the current aim of forestry to increase the vertical integration between forest management and the individual utilization of round wood. Use and value of trees depend highly on diameter. Accordingly, management plans or silvicultural operations are not only designed to initiate quality and volume production but also to maintain desirable diameter distribution. Diameter distribution is also important for growth and volume prediction. Besides, knowledge of diameter distribution facilitates a suitable sampling design for growing stock estimation (Jayaraman and Rugmini, 1988). In general, diameter distribution gives a clear insight into the structure of the forest and is an important basis for economic decisions and may be used in planning, if actual distributions can be successfully predicted via mathematical distribution functions (Loetsch et al., 1973). Such role of the tree diameter (breast height) distribution has initiated many researchers to model diameter distributions.

Use of probability density function is the widely used method for describing tree diameter distributions. To date, many probability density functions are investigated for the propose. These include Gamma (Nelson, 1964; Laar, 1990 ; Schrender and Swank, 1974; Swindel et al., 1987), Log-normal (Bliss and Reinker, 1964), Normal(Schrender and Swank, 1974), Beta (Loetsch, et al., 1973; Burkhart and

Strub, 1974; Laar, 1990; Swindel et al., 1987; Jayaraman and Rugmini, 1988; Maltamo et al., 1995), Weibull (Bailey and Dell, 1973; Schrender and Swank, 1974; Laar, 1990; Little, 1983; Magnussen, 1986; Kikki and Päivinen, 1986; Swindel et al., 1987; Ueno and Osawa, 1987; Jayaraman and Rugmini, 1988; Kilkki, et al., 1989; Holte, 1993; Maltamo et al., 1995; Maltamo, 1997; Lindsay et al., 1996), and Johnson's system b (Holte, 1993; Zhou and Mctague, 1996).

For the last two decades, particularly for even aged diameter distributions, the case in many studies, Weibull enjoyed the most popularity. In this study, the two-parameter Lognormal, and Weibull distribution are investigated and compared for modelling the tree diameter distribution under study in plot level. The fact that the sample is taken from the portion of the population with diameter at breast height (dbh) greater or equal to 5cm, it is a left truncated data. Accordingly, since failure to account for truncation can lead to biased inference (Kalbfleish and Lawless, 1992), the truncated candidate distributions are employed for parameter estimation. The performance of the fit of the distributions is compared with chi-square test of Goodness of fit.

In general, the likelihood function for a random sample of n observations drawn from a left truncated random variable X at a point T is

$$L(\; ; \mathbf{x}) = \prod_{i=1}^{n} \frac{f(x_i;)}{1-F(T)} \qquad (4)$$

where $x_i \geq T$, $f(x_i;)$ is the probability density function (pdf) and $F(T)$ is the cumulative distribution function (cdf) at the truncated point T. From (4), it is possible to form the likelihood functions of the distributions concerned and derive the parameter estimators using maximum likelihood method.

4.2.1.1 **Lognormal distribution**

The two parameter lognormal distribution can be defined as the distribution of a random variable X, whose logarithm is normally distributed. The probability density function of X is

$$f(x;\mu,\sigma^2) = (2\pi)^{-1/2}(\sigma x)^{-1}\exp(-(\ln(x)-\mu)^2/2\sigma^2),\; x>0,\ \sigma>0$$
$$= 0, \text{ otherwise} \qquad (5)$$

The lognormal distribution is generally identified as a heavily tailed distribution that is unlikely to be useful for small skewness, $\alpha < 1$, (Cohen, 1988).

The parameter estimation for the truncated lognormal distribution is commonly carried out by exploiting its relation to Normal distribution. If the random variable X has a lognormal distribution (μ, σ^2), then Y = ln(X) has a normal distribution (μ, σ^2). Consequently, to analyze the sample data from a lognormal distribution, it is only needed the logarithmic transformation of the data and to adopt applicable normal theory for the rest of analysis (or parameter estimation). In this case, the transformation $y_i = \ln(x_i)$ is made to the sample data and then a left truncated normal distribution is used for parameter estimation.

From equation (4) the likelihood function for the transformed sample data (normally distributed) is

$$L(\mu, \sigma^2; y) = (1 - F(\xi))^{-n} \, (\sigma\sqrt{2\pi})^{-n} \exp\left[-\sum_{i=1}^{n}(y_i - \mu)^2 / 2\sigma^2\right] \qquad (6)$$

where $F(\xi) = \int_{-\infty}^{\xi} \phi(t)dt$, $\xi = (T - \mu)/\sigma$, T = truncation point and

$$\phi(t) = (\sqrt{2\pi})^{-1} \exp(-t^2/2) .$$

Using the common method for derivation of the maximum likelihood estimators, Cohen(1959) has derived the estimators from equation (6) as:

$$\hat{\sigma}^2 = s^2 + \hat{\theta}\,(\bar{y} - T)^2, \text{ and}$$

$$\hat{\mu} = \bar{y} - \hat{\theta}\,(\bar{y} - T) \qquad (7)$$

where $\hat{\theta} = \theta(\hat{\xi}) = \dfrac{Z(\xi)}{Z(\xi) - \xi}$, $Z(\xi) = \dfrac{\phi(\xi)}{1 - F(\xi)}$ and, s^2 and $\bar{y}$ are sample variance and mean, respectively. The same estimators can also be derived using moment method (see Cohen, 1991).

To compute the estimates from (7) manually a prior estimate of $\hat{\theta}$ is required from tables made for this purpose (see Cohen 1959, Cohen 1961, Cohen 1991). However in this study a Fortran program is written to carry out the parameter estimation by directly maximizing the loglikelihood function.

4.2.1.2 **Weibull distribution**

A probability density function (pdf) of the Weibull distribution may be derived from the standard exponential distribution with pdf

$$f(y) = e^{-y}, \ y>0 \tag{8}$$
$$= 0 \text{ , otherwise.}$$

With $Y = \left[\frac{X}{\delta}\right]^{\beta}$ in (8), the transformation method gives that the random variable X has a Weibull distribution with pdf

$$f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1}\exp\left(-\left[\frac{X}{\delta}\right]^{\beta}\right), \ x \geq 0, \ \beta > 0, \ \delta > 0$$
$$= 0, \text{ otherwise.} \tag{9}$$

Reparameterization of the scale parameter to $\alpha = \delta^{-\beta}$ gives Weibull cumulative distribution function (cdf), and pdf, respectively,

$$F(x) = 1-\exp(-\alpha \ x^{\beta}), \quad \text{and} \tag{10}$$
$$f(x) = \alpha \ \beta \ x^{\beta-1}\exp(-\alpha \ x^{\beta}) \tag{11}$$

where $x \geq 0, \ \beta > 0, \ \alpha > 0$.

Introducing α as a scale parameter provides easier derivation of the estimators.

The corresponding cdf and pdf for the left truncated Weibull distribution (LTWD) at a point T are, respectively,

$$F(x) = 1-\exp\left[-\alpha\left(x^{\beta}-T^{\beta}\right)\right] \tag{12}$$
$$f(x) = \alpha \ \beta \ x^{\beta-1}\exp\left[-\alpha\left(x^{\beta}-T^{\beta}\right)\right], \text{ where } x>T>0. \tag{13}$$

On the basis of that maximum likelihood estimation method will be used for parameter estimation, an effort is exerted below to show the derivation of the estimators.

The likelihood function of LTWD(13) gives

$$L(\alpha, \beta; \mathbf{x}) = \prod_{i=1}^{n}\left(\alpha \ \beta \ x_i^{\beta-1}\right)\exp\left[-\sum_{i=1}^{n}\alpha\left(x_i^{\beta}-T^{\beta}\right)\right], x_i > T, \beta > 0, \alpha > 0 \tag{14}$$

The loglikelihood function of (14) can be written as

$$L(\alpha, \beta; x) = n \ln(\alpha \beta) + (\beta - 1) \sum_{i=1}^{n} \ln(x_i) - \sum_{i=1}^{n} \alpha (x_i^{\beta} - T^{\beta})$$

where $x_i > T, \beta > 0, \alpha > 0$. (15)

Carrying out derivatives with respect to the scale and shape parameters in (15) and equating them to zero gives the following system of equations:

$$\partial L / \partial \alpha = \frac{n}{\alpha} - \sum_{i=1}^{n} (x_i^{\beta} - T^{\beta}) = 0 \quad (16)$$

$$\partial L / \partial \beta = \frac{n}{\beta} + \sum_{i=1}^{n} \ln x_i - \alpha \sum_{i=1}^{n} (x_i^{\beta} \ln x_i - T^{\beta} \ln T) = 0 \quad (17)$$

From (16) $\hat{\alpha} = \dfrac{n}{\sum (x_i^{\beta} - T^{\beta})}$. (18)

Substituting (18) into (15) and (17), yields

$$L(\beta) = n(\ln n - 1) + n \ln \beta + (\beta - 1) \sum_{i=1}^{n} \ln x_i - n \ln \left(\sum_{i=1}^{n} (x_i^{\beta} - T^{\beta})\right), \text{ and} \quad (19)$$

$$\frac{n}{\beta} + \sum_{i=1}^{n} \ln x_i - \frac{n \sum_{i}^{n} (x_i^{\beta} \ln(x_i) - T^{\beta} \ln T)}{\sum_{i}^{n} (x_i^{\beta} - T^{\beta})} = 0 \quad (20)$$

The MLE $\hat{\beta}$ may be obtained by iteration of (20) or by finding the global maximizer of the one dimensional optimization of (19). The latter approach is used in this work using Fortran programming language. The optimization is one dimensional that maximizes $L(\beta)$, $\beta \in (a, b)$, where $0<a<b$ (Wingo, 1989).

Wingo noted that locally convergent root finders such as Newton's method may require modifications to ensure computer floating point overflow does not occur when evaluation of χ_i^{β} or T^{β} is attempted.

4.2.2 **Spatial continuity**

The effort of this work is to examine the spatial continuity of tree diameter distribution with both parametric and non-parametric approaches. Using the selected Weibull distribution, parameters are estimated for each plot. Consequently, investigation for spatial variation of the parameters is considered. Since the parameters are expected to reflect the dbh distribution at each plot, such study is an attempt to investigate whether tree dbh distribution exhibits spatial continuity or not.

Normally, in the non-parametric approach, several cutoffs of the cumulative distribution function of the diameter are required to construct spatial continuity functions that may describe the diameter distribution. However for a demonstration purpose single cutoff is selected by considering its economic importance and range of diameter distribution of the compartment. These cutoffs in cm are 25, 20, 25, 20, 18, and 18 respectively in compartment 17, 34, 39, 41, 43 and 45. Accordingly, the cumulative distribution function of the cutoff is computed from each plot in the compartment under consideration and used to study the spatial variation in tree diameter distribution by dealing with omnidirectional semivariograms and models therein.

Both the non-linear least squares (NLS) and weighted least squares, Cressie(1985) (WLS) parameter estimation methods are used when model estimation is deemed necessary. However, in the cross validation study the parameters estimated by WLS is used for almost all Kriging estimation.

To assess the fitted variogram models, the use of the standardized prediction residuals (SPR) is employed as defined in equations (1) and (2). The stem and leaf plot of the SPR is also used to check outliers. A quantile-quantile plot and linear regression are also used to see the relation between the kriged and the observed values.

All Geostatistical computations in this work are made on S+ Spatialstats version 3.3 software statistical programme.

5. Results

5.1 Goodness of fit

For the reasons stated in the methodology, truncated distributions are considered appropriate for fit of the diameter distribution under study. To further strength the argument, a comparison of fit of Weibull parameters estimated from truncated and untruncated distributions is made. To this end, materials of Compartment 45 are considered for demonstration. From compartment 45, the parameters of the truncated and untruncated Weibull distributions are estimated using maximum likelihood estimation method. The parameter estimates of the distributions and their corresponding computed and tabular values of χ^2 as measure of fit is contained in Table 2.

Table 2: Parameter estimates and χ^2 values for truncated and untruncated Weibull distribution

Data	Trees	Truncated			Untruncated			Table
Comp	(N)	Scale	Shape	χ^2	Scale	Shape	χ^2	$\chi^2_{.05,20}$
45	814	13.1278	2.8585	26.7	13.6457	3.2582	69.1	31.4

The connected plots of the fit of these estimates are shown in Fig 1. The fit of the truncated estimates is very much better as observed from Fig 1.

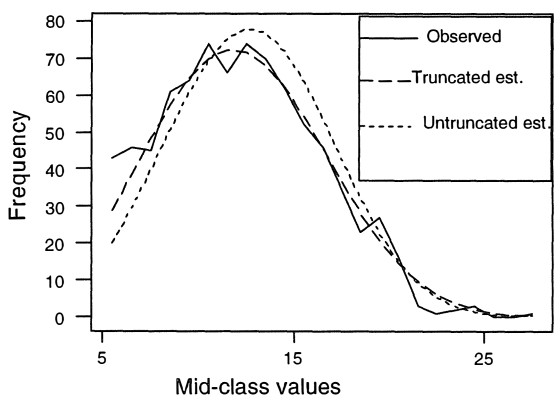


Fig 1: Comparison of fit of truncated and untruncated Weibull distributions

It is also a good precaution to note that according to the chi-square(χ^2) test at 5% level of significance, the fit of the parameter estimates from the truncated distribution is accepted while the other not. Therefore, an attempt to overlook truncated data and to assume inference from untruncated (complete) distribution may end in wrong conclusion.

Accordingly, on the basis of the aforementioned argument, the parameters of both left truncated Lognormal and Weibull distributions are estimated at each plot in a compartment with the methods demonstrated in section 4.2.1.1 and 4.2.1.2, respectively. Comparison of Goodness of fit between the distributions is made using χ^2 test. The parameter estimates of each plot by compartment for both Lognormal and Weibull distributions with their corresponding χ^2 as a measure of Goodness of fit is shown in Appendix 2. For the computation of the χ^2, classes ranging from 3-9 depending on the number of trees in the plot and with about equal number of trees are formed. χ^2 is not computed for plots with number of trees less than 15. With exception to compartment 39 with 7 plots and compartment 17 with one plot, all other plots have at least 15 trees. See also Appendix 2 for the number of trees in each plot.

Table 3 indicates acceptance or rejection of fit of the performance of the distributions at 5% level of significance. In general both are accepted for fitting the diameter distribution under study.

Table 4 compares the Goodness of fit of the distributions. With exception to compartment 43, the computed χ^2 from Weibull fit is smaller in most plots of the other compartments as compared to Lognormal. Hence, on average the summary in Table 4 favours Weibull distribution as a better fit. Accordingly, the Weibull parameter estimates are used in the ensuing study of the spatial continuity.

Table 3: Fit of plot data to Lognormal and Weibull as measured by χ^2 at 5% significance level.

	Lognormal Number of plots			Weibull Number of plots		
Comp.	Accepted	Rejected	Others	Accepted	Rejected	Others
17	24	3	12	26	1	12
34	24	-	-	24	-	-
39	18	2	16	18	2	16
41	15	-	1	15	-	1
43	36	1	-	35	3	-
45	22	-	-	22	-	-

Others = Plots with trees less than 15 or plots with three classes, making degree of freedom (df) zero.

Table 4: Comparison of the fit of the Lognormal and Weibull distributions for the plots in each compartment

	Number of plots show small χ^2	
Comp.	Lognormal	Weibull
17	10 (26.3 %)	28 (73.7%)
34	6 (25%)	18 (75%)
39	7 (26%)	20 (74%)
41	7 (44%)	9 (56%)
43	21 (55%)	17 (45%)
45	8 (36%)	14 (64%)

5.2 **Parametric approach for describing spatial continuity of tree diameter distribution.**

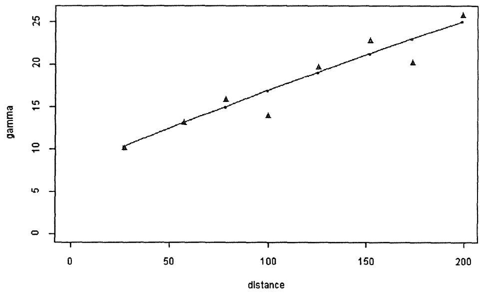
A study of semivariogram and corresponding models are carried out taking each scale and shape parameter estimate of the Weibull distribution at each plot as a diameter distribution attribute. The variogram models and /or variograms for scale and shape parameter estimates of the six compartments are displayed in Fig 2 and Fig 3, respectively. The parameter estimates of the models are given in Table 5. The variograms, in general, seem to show that both Weibull parameter estimates exhibit spatial continuity which may connotes that diameter distribution also possesses spatial continuity in its nature.

Kriging estimation for cross validation study is made in the compartments were variogram models are displayed. The variogram models estimated with WLS (Cressie, 1985) is used for kriging. To obtain a diagnostic check for the fit of the variogram models presented, the standardized predicted residuals (SPR) are used. The stem and leaf plot of the SPR show no potential outliers (see appendix 3). The mean (M) and root-mean-square (RMS) of the SPR as defined in equations (1) and (2) are computed and presented in Table 6. The statistics M and RMS are approximately close to 0 and 1, respectively for all models. This result may indicate fairness of the models and the kriging error but it can not be a proof.

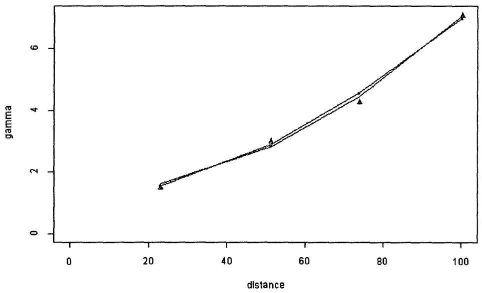
Fig 4 indicates the plots of the kriged estimates versus the Weibull parameter estimates. The regression equation of the two variables with the adjusted r^2, the kriged estimate as explanatory variable, is also presented under respective plots. The coefficient of correlation is significant at 5% level and in most compartments fairly high. A quantile-quantile(qq) plots of the kriged estimates versus the Weibull estimates are also displayed in Fig 5. The qq plots indicate the similarity of the distributions since the plots are clustered around the line x = y where distributions are said to be identical.

Thus, the cross validation study seem to support that, in general, diameter distribution at compartment level is spatially dependent attribute or a regionalized variable in the terms of Geostatistics.

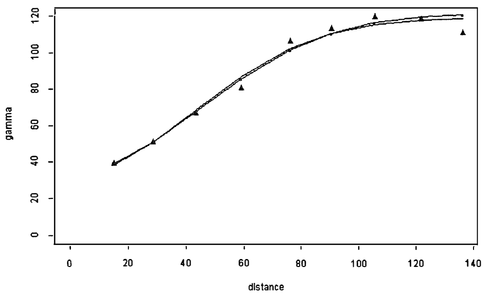
Compartment 17
Exponential model



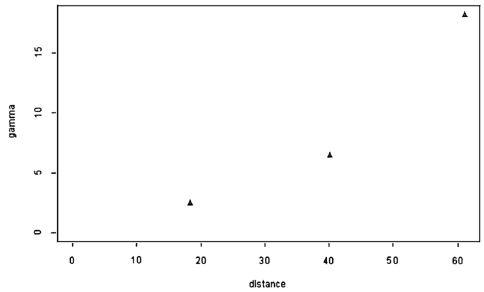
Compartment 34
Gaussian model



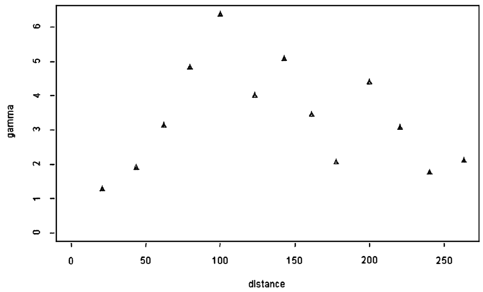
Compartment 39
Gaussian model



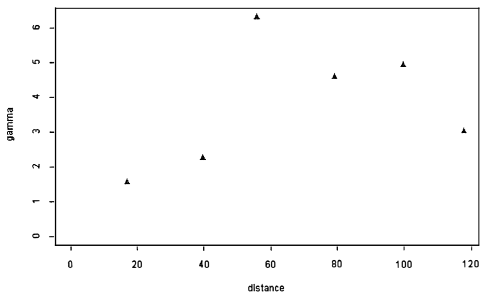
Compartment 41



Compartment 43



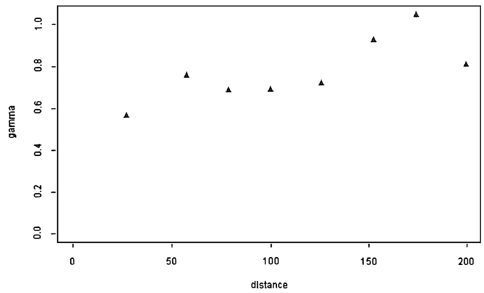
Compartment 45



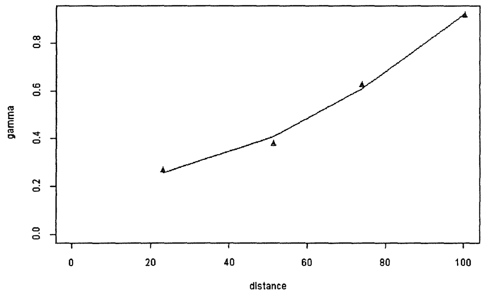
legend:
WLS
NLS

Fig 2: Variogram models and/or variograms of Weibull scale(δ) parameter estimates

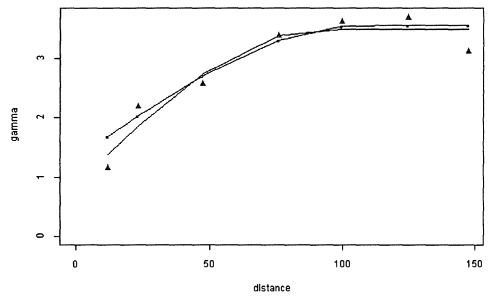
Compartment 17



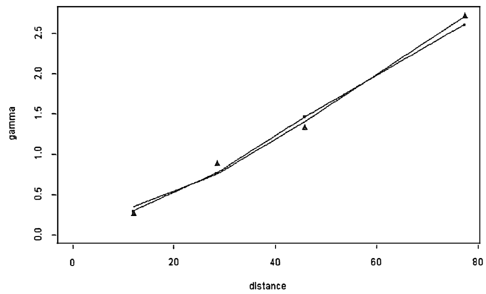
Compartment 34
Gaussian model



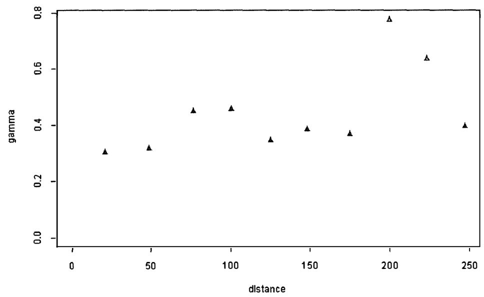
Compartment 39
Spherical model



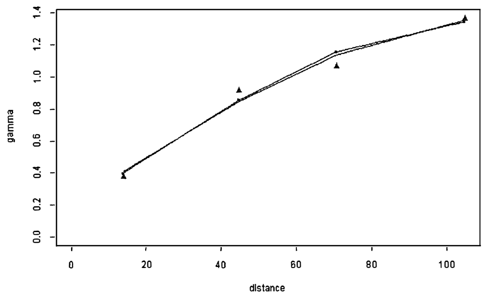
Compartment 41
Gaussian model



Compartment 43



Compartment 45
Spherical model



legend:
WLS
NLS

Fig 3: Variogram models and/or variograms of Weibull shape(β)parameter estimates.

Table 5: Parameter estimates of the Variogram models (for Weibull estimates) displayed in Fig 2& 3.

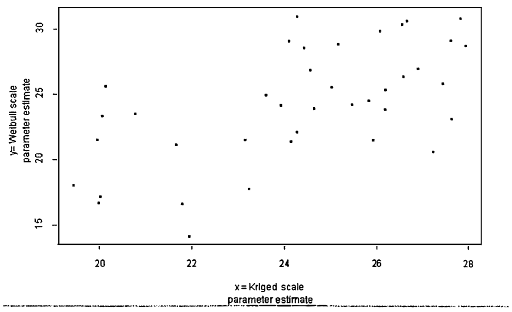
			NLS method			WLS method		
Comp	Variable	Model	Nugget	Sill	Range	Nugget	Sill	Range
17	Scale	Exp	-	-	-	7.745	83.403	855.657
34	Scale	Gauss	1.308	312.411	735.992	1.205	20.999	177.031
39	Scale	Gauss	32.753	86.532	59.482	34.453	87.338	62.916
34	shape	Gauss	0.218	7.009	308.769	-	-	-
39	Shape	Spher	0.863	2.616	90.398	1.306	2.239	106.238
41	Shape	Gauss	0.259	3.924	77.860	0.189	3.116	62.925
45	Shape	Spher	0.199	1.169	115.961	0.179	1.168	109.007

* The blanks indicate that the program aborted to estimate the parameters.

Table 6 : The mean (M) and root-mean-square (RMS) of the standardized predicted residuals

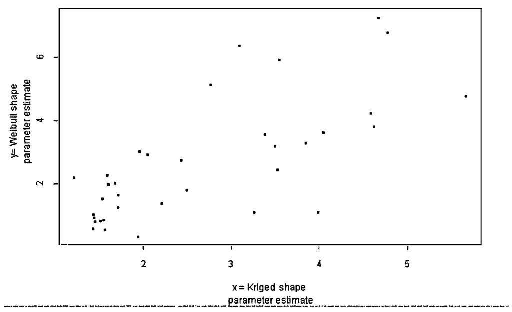
			Statistics of SPR	
Comp.	Variable	Model	M	RMS
17	Scale	Exp	-0.005	0.971
34	Scale	Gauss	0.0140	1.254
39	Scale	Gauss	0.007	0.883
34	shape	Gauss	-0.0002	1.059
39	Shape	Spher	0.006	0.893
41	Shape	Gauss	-0.019	0.998
45	Shape	Spher	-0.031	0.997

Compartment 17



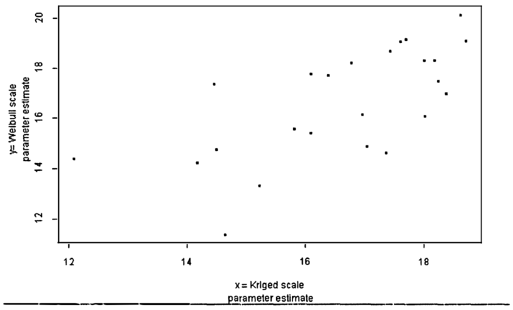
$y = -0.982+1.039x$, adjusted $r^2 = 0.36$

Compartment 39



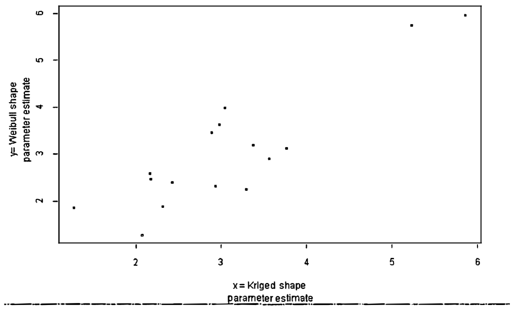
$y = -0.178 + 1.074x$, adjusted $r^2 = 0.49$

Compartment 34



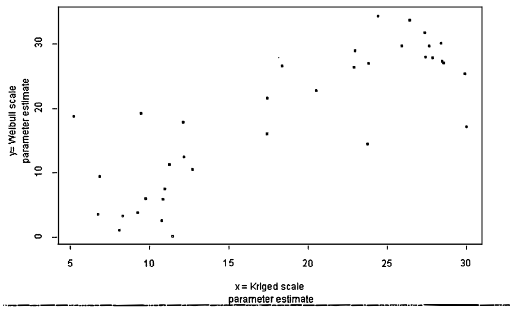
$y = 2.456+0.854x$, adjusted $r^2 =0.42$

Compartment 41



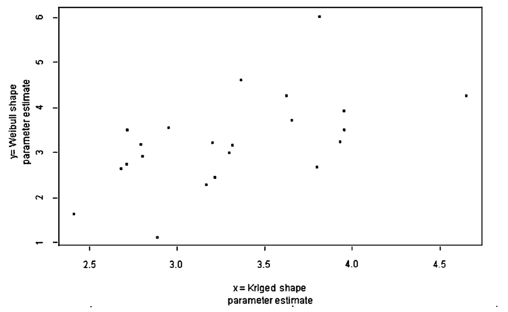
$y = 0.0256 + 0.985x$, adjusted $r^2 = 0.76$

Compartment 39



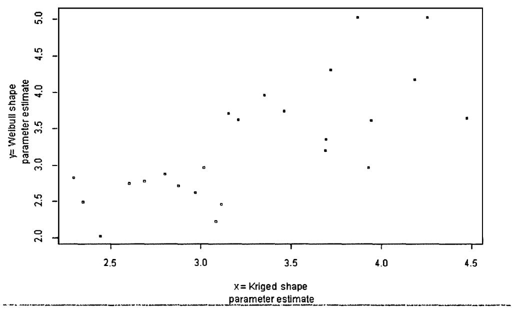
$y = -0.926 + 1.056x$,adjusted $r^2 = 0.67$

Compartment 45



$y = -0.193 + 1.042 x$, adjusted $r^2 = 0.29$

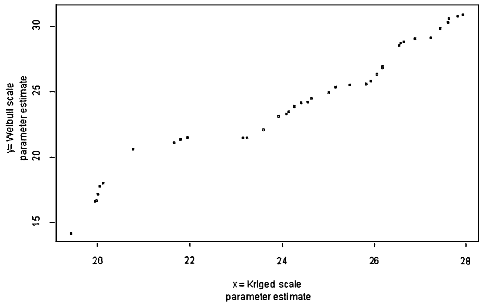
Compartment 34



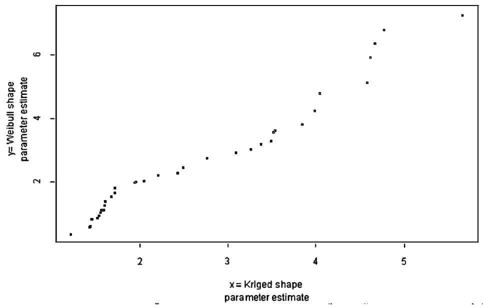
$y = 0.197 + 0.940 x$, adjusted $r^2 = 0.51$

Fig 4: Plots of the kriged estimates vs Weibull estimates.

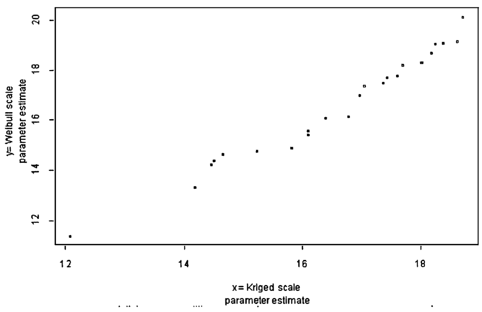
Compartment 17



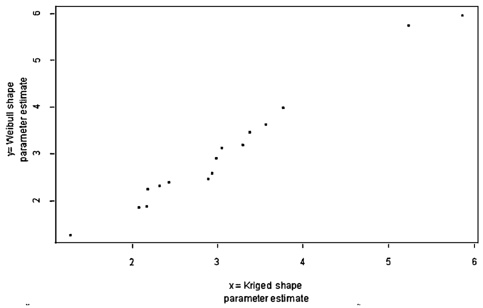
Compartment 39



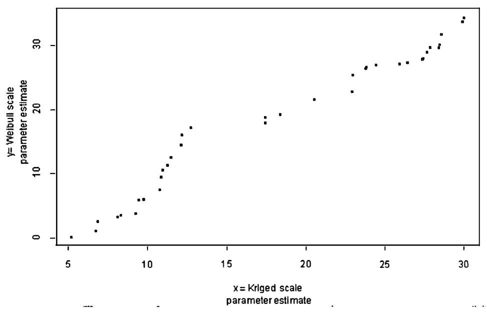
Compartment 34



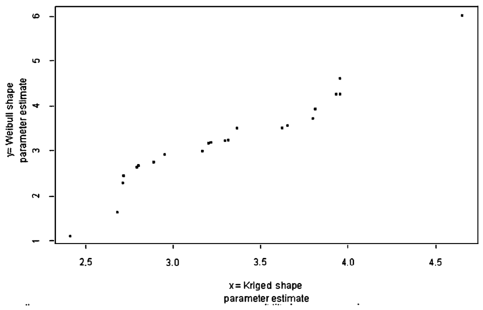
Compartment 41



Compartment 39



Compartment 45



Compartment 34

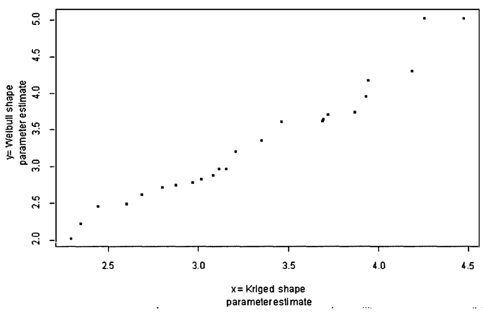


Fig 4: A quantile-quantile plots of the kriged estimates vs Weibull estimates.

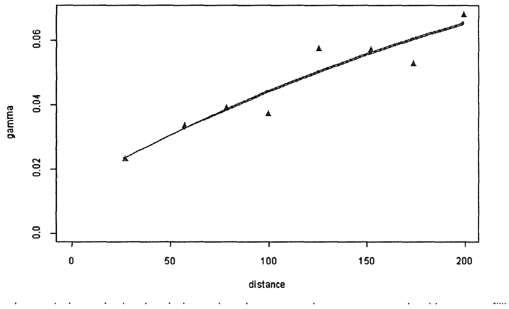
5.3 Non-parametric approach for describing spatial continuity of tree diameter distribution.

This non-parametric approach for studying spatial continuity of diameter distribution refers to the computation of cumulative distribution function (cdf) at several cutoffs and investigation of spatial continuity functions for each cutoff. For most practical problems, this approach of describing spatial continuity of regionalized variables may require to determine few cutoffs that are of economic or any other interest. Then it will be of particular interest to study the continuity functions of these cutoffs. In light of this argument and range of distribution of the diameter in each compartment, a single cutoff is considered in this study to investigate spatial dependency in diameter distribution. The variogram model and/or variogram of the cdf of diameter at the selected cutoff in each compartment is shown in Fig 6. The functions and parameter estimates of the variogram models displayed in Fig 6 are contained in Table 7. The variograms of the considered cutoffs in all compartments indicate spatial dependency of the diameter distribution varying in scale and range from compartment to compartment. The models fitted to the variogram estimates also differ with compartment.

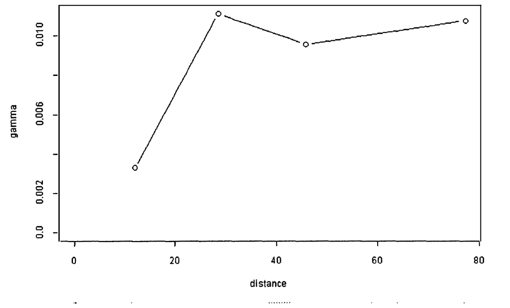
As it was in the parametric case, the standardized predicted residuals (SPR) are used to carry out diagnostic check of the fitted models. The stem and leaf plots show no warranty to outliers (see appendix 3). The mean (M) and root-mean-square(RMS) of the SPR are computed as shown in Table 8. The evidence shows M and RMS are fairly close to 0 and 1, respectively.

The plots of the kriged cdf estimates versus the observed cdf of dbh with their corresponding regression functions are presented in Fig 7. Fig 8 shows the quantile-quantile plot of the kriged versus the observed cdf at the selected cutoffs. In general speaking, the cross validation study seem to suggest the spatial dependency of diameter distribution which agrees to the conclusion obtained from the parametric approach study.

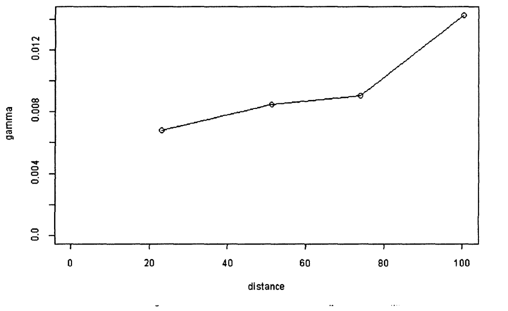
Compartment 17
cdf cutoff = 25 cm
Exponential model



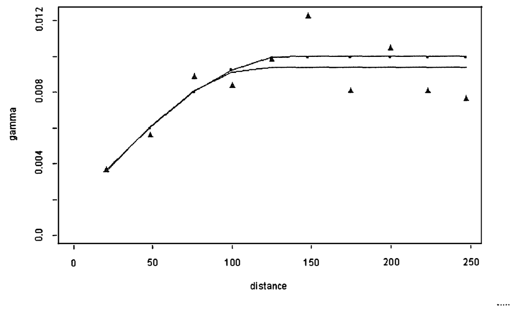
Compartment 41
cdf cutoff = 20 cm



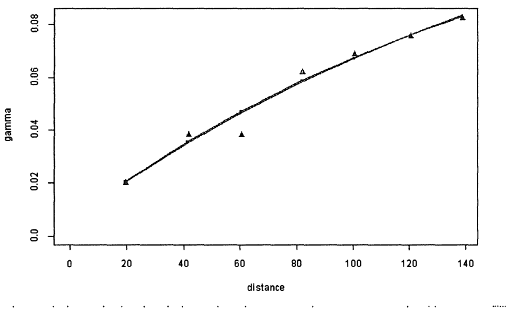
Compartment 34
cdf cutoff = 20cm



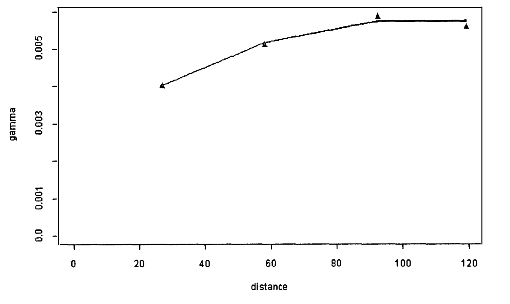
Compartment 43
cdf cutoff = 18 cm
Spherical model



Compartment 39
cdf cutoff = 25 cm
Exponential model



Compartment 45
cdf cutoff =18 cm
Spherical model



legend:

WLS

NLS

Fig 6: Variogram models and/or variograms of cumulative distribution function (cdf) of dbh at selected cutoffs.

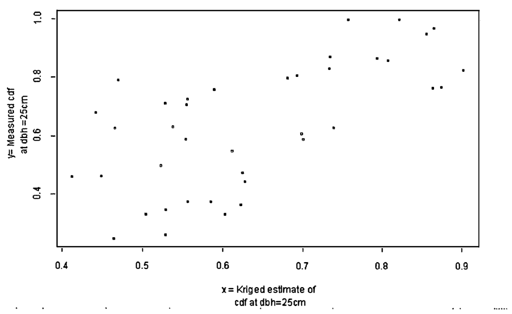
Table 7: Parameter estimates of the variogram models of cdf displayed in Fig 6

	cdf		NLS method			WLS method		
Comp.	cutoff	Models	Nugget	Sill	Range	Nugget	Sill	Range
17	25 cm	Exp	0.015	0.110	328.849	0.0145	0.104	294.418
39	25 cm	Exp	0.006	0.153	195.442	0.0056	0.138	169.043
43	18 cm	Spher	0.002	0.008	118.305	0.002	0.008	133.607
45	18 cm	Spher	0.003	0.003	95.062	0.003	0.003	96.703

Table 8: The statistics of the Standardized predicted residuals (SPR) from the models estimated by WLS as shown in Table 7

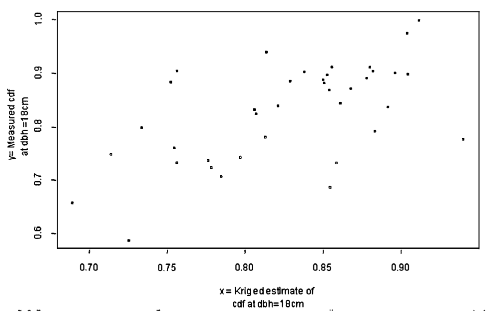
	cdf		Statistics of SPR	
Comp.	cutoff	Models	M	RMS
17	25 cm	Exp	0.003	0.954
39	25 cm	Exp	-0.006	0.975
43	18 cm	Spher	0.011	1.032
45	18 cm	Spher	-0.002	0.977

Compartment 17



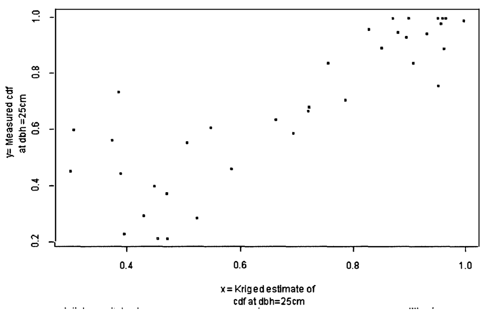
$y = -0.012 + 1.021\,x$,
adjusted $r^2 = 0.42$

Compartment 43



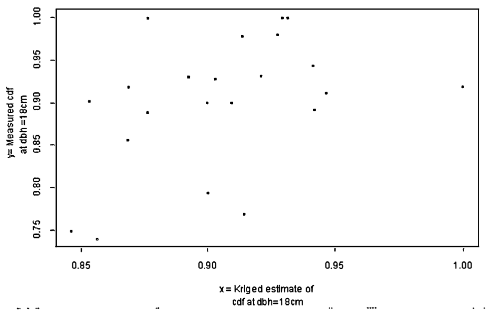
$y = 0.104 + 0.876x$,
adjusted $r^2 = 0.32$

Compartment 39



$y = 0.014 + 0.977x$,
adjusted $r^2 = 0.72$

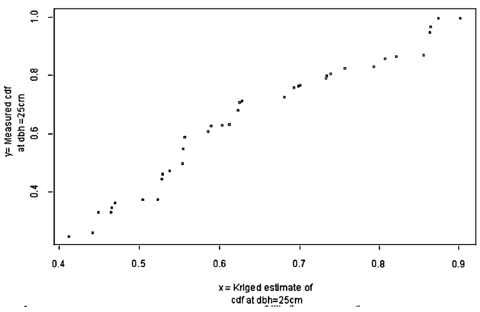
Compartment 45



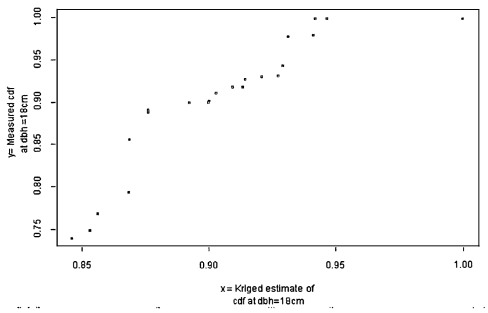
$y = 0.105 + 0.88x$
adjusted $r^2 = 0.13$

Fig 7: Plots of the kriged cumulative distribution function (cdf) estimates vs the measured cdf of dbh.

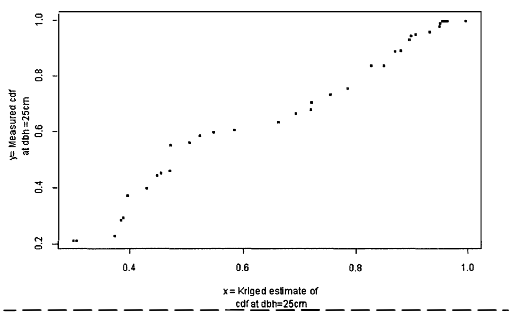
Compartment 17



Compartment 45



Compartment 39



Compartment 43

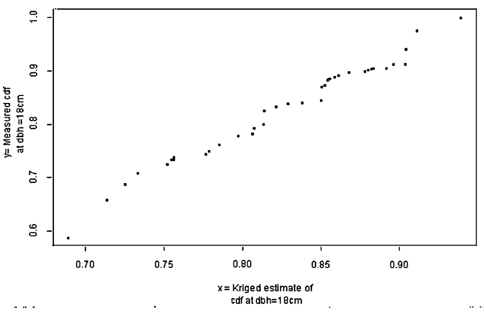


Fig 8: A quantile-quantile plots of the kriged cumulative distribution function (cdf) estimates vs the measured cdf of dbh.

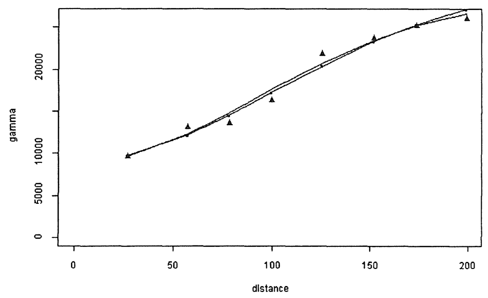
5.4 Spatial continuity of number of stems per hectare

In this section an effort is made to investigate the spatial dependency of number of stems per hectare greater or equal to selected cutoff using isotropic semivariogram and corresponding model. The same cutoffs used for the cdf spatial continuity study (5.3) are also used here. The variogram models and/or variograms of the number of stems per hectare greater or equal to selected cutoff in each compartment is presented in Fig 9. The functions and parameter estimates of the variogram models displayed in Fig 9 is shown in Table 9.

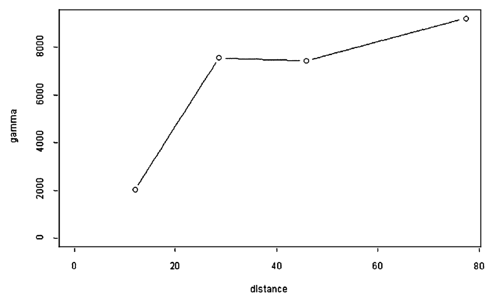
The cross validation study followed by diagnostic check for the models using SPR is carried out. After the stem and leaf plot of the SPR is checked for outliers (see appendix 3), the statistics of the SPR are computed as presented in Table 10. Table 10 seem to show no evidence against the fitted models and their corresponding kriging errors since M and RMS are approximately 0 and 1, respectively.

Fig 10 shows plots of the kriged versus the observed number of stems per hectare greater or equal to the selected cutoff at each compartment. The corresponding regression function of the variables, the kriged values as explanatory variable, is also presented along with the adjusted r^2. A quantile-quantile plot of the kriged versus the observed number of stems is displayed in Fig 11. In general, the presented evidence seem to depict a considerable spatial dependency of the number of stems greater or equal to the selected cutoff (per hectare) in each compartment.

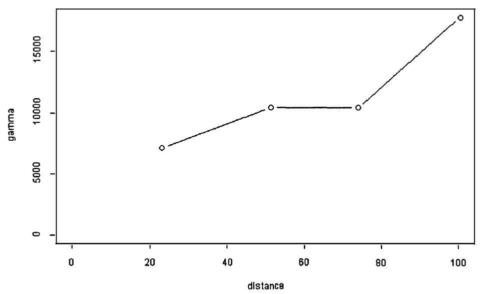
Compartment 17
cutoff = 25cm
Gaussian model



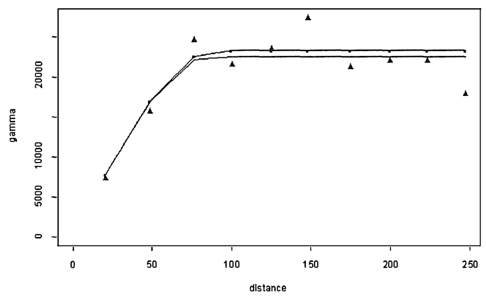
Compartment 41
cutoff = 20cm



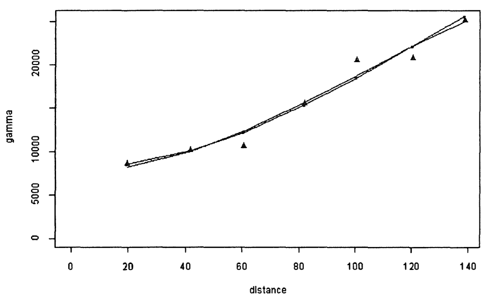
Compartment 34
cutoff = 20 cm



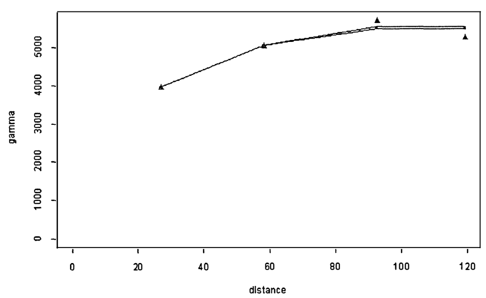
Compartment 43
cutoff = 18cm
Spherical model



Compartment 39
cutoff = 25cm
Gaussian model



Compartment 45
cutoff =18cm
Spherical model



legend:
WLS
NLS

Fig 9: Variogram models and/or variograms for number of stems per hectare with diameter at breast height (dbh) greater or equal to the selected cutoff.

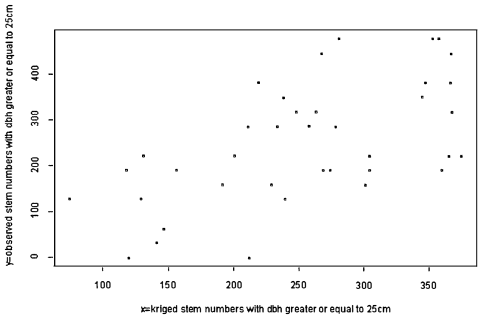
Table 9: Functions and parameter estimates of the variogram models for number of stems displayed in Fig 9

	stems		NLS method			WLS method		
Comp.	cutoff	Model	Nugget	Sill	Range	Nugget	Sill	Range
17	25 cm	Gauss	8851.275	19656.77	130.132	9025.394	21125.24	142.6715
39	25 cm	Gauss	7632.553	28445.59	142.512	8021.378	40465.02	183.2733
43	18 cm	Spher	0	22500	86.2	0	23286.648	89.7
45	18 cm	Spher	2781.05	2726.53	88.780	2800.394	2759.19	91.48792

Table 10: The statistics of SPR for the models estimated by WLS as shown in Table 9.

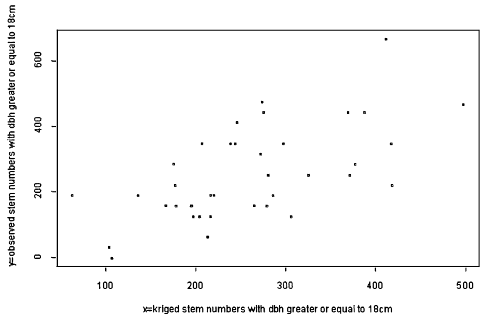
	stems		Statistics of SPR	
Comp.	cutoff	Model	M	RMS
17	25 cm	Gauss	-0.007	0.976
39	25 cm	Gauss	0.003	0.951
43	18 cm	Spher	-0.017	1.027
45	18 cm	Spher	-0.001	0.934

Compartment 17



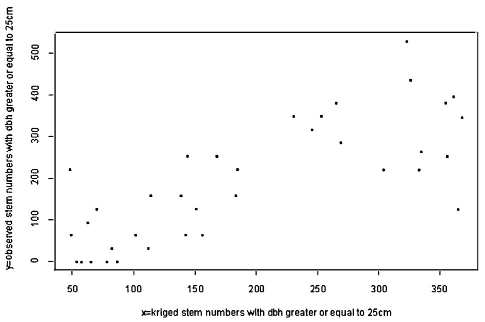
$y = 10.333 + 0.954x$,
adjusted $r^2 = 0.36$

Compartment 43



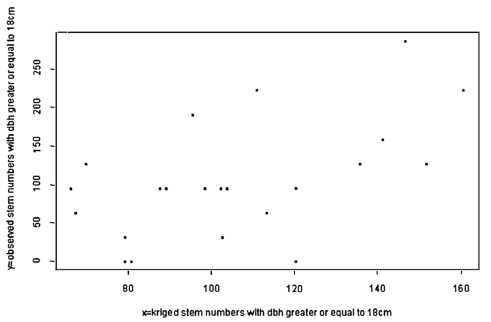
$y = 19.421 + 0.912x$
adjusted $r^2 = 0.38$

Compartment 39



$y = 2.490 + 0.990x$
adjusted $r^2 = 0.58$

Compartment 45

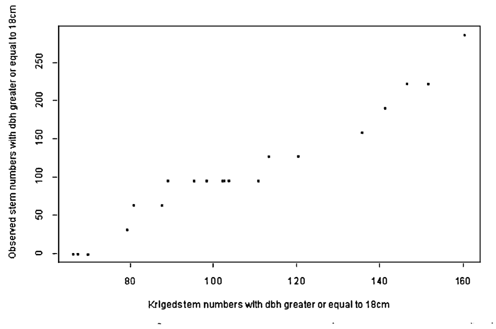


$y = -49.80 + 1.469x$,
adjusted $r^2 = 0.26$

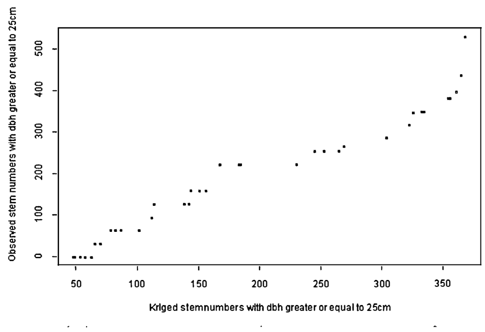
Fig 10: Plots of the kriged vs the observed number of stems per hectare greater or equal to the selected diameter at breast height (dbh)

Compartment 17

Compartment 45



Compartment 39



Compartment 43

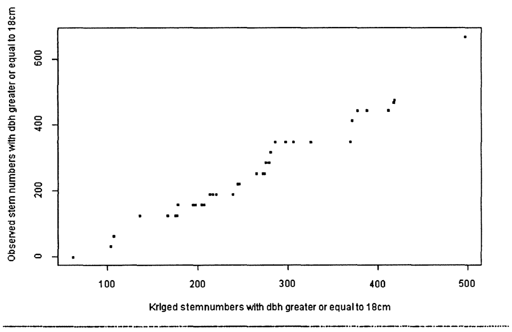


Fig 11: A quantile-quantile plots of the kriged vs the observed number of stems per hectare greater or equal to the selected diameter at breast height (dbh).

5.5. **Practical aspect**

Kriging is unbiased and weighted linear combination estimation, where, in general, more weight is given to samples closer to the prediction location. Hence, variogram models used for kriging may seem very much promising when samples considered for kriging are very closer to the prediction location. But in practical inventory work, models may be imposed to estimate by kriging sample values located at a reasonably far away from the estimation location, say at least 25 meters. In such cases, it is of practical importance to investigate the performance of the models. Accordingly, an effort is made to compare predictions obtained from the cross validation study and predictions made by kriging samples at least 25 meters distance away from estimation location.

As it was the case in the predictions obtained by cross validation approach, the standardized predicted residuals (SPR) are used to assess the fitted models performance when the said restriction is put to the samples. Before the statistics of SPR is computed, the stem and leaf plot is checked for outliers (see Appendix 4). The statistics of SPR is shown in Table 11. From Table 11, it seems that variability of the estimates has increased as the RMS of most of the variables has exceeded one. Otherwise, it seems not apparent to count other indications against the fitted models (or predicted values) as far as the statistics of SPR is concerned since M and RMS are fairly close to 0 and 1, respectively.

The prediction obtained from cross validation (P_1) and distance restriction imposed samples (P_2) are compared by their sum of the squares of residuals (SSR). Table 12 compares the SSR of the two prediction types. The column (P_2/P_1 -1)% shows the increase of SSR in percent due to the distance restriction imposed on the samples. In more than 50% of the variables in the study, the SSR increased by more than 50% due to the distance restriction put on samples used for kriging. Therefore, it seems that discrepancy increases to considerable amount when samples are 25 meters or more distance away from the prediction location in this study.

Table 11: The Statistics of SPR when kriging is made from observations at least 25 meters away from the prediction location

a) Weibull parameters

			Statistics of SPR	
Comp.	Variable	Model	M	RMS
17	Scale	Exp	-0.059	1.015
34	Scale	Gauss	0.0156	1.230
39	Scale	Gauss	-0.0137	0.959
34	shape	Gauss	0.006	1.086
39	Shape	Spher	-0.024	0.877
41	Shape	Gauss	0.006	1.082
45	Shape	Spher	-0.018	1.131

b) Cumulative distribution function (cdf) of diameter

	cdf		Statistics of SPR	
Comp.	cutoff	Models	M	RMS
17	25 cm	Exp	0.090	1.010
39	25 cm	Exp	-0.012	0.921
43	18 cm	Spher	-0.035	1.092
45	18 cm	Spher	0.007	1.198

c) Stem per hectare greater or equal to the given cut-off.

	Stems		Statistics of SPR	
Comp.	cutoff	Model	M	RMS
17	25 cm	Gauss	-0.054	0.968
39	25 cm	Gauss	0.014	0.996
43	18 cm	Spher	0.036	1.077
45	18 cm	Spher	-0.008	1.165

Table 12: Comparison of the SSR when predictions are obtained from cross validation (P_1) and distance restriction imposed samples (P_2)

Comp	Variables/ Weibull parameters	SSR (P_1)	SSR (P_2)	ratio (P_2/P_1)	$(P_2/P_1 - 1)\%$
17	cdf cutoff = 25cm	1.008214	1.320211	1.309455	30.95
39	cdf cutoff = 25cm	0.665583	1.028282	1.544934	54.49
43	cdf cutoff = 18cm	0.20868	0.335349	1.607001	60.70
45	cdf cutoff = 18cm	0.098292	0.171486	1.744659	74.47
45	stem per ha ≥ 18cm	88220.24	159496	1.80793	80.79
43	stem per ha ≥ 18cm	456779.4	895718.4	1.960943	96.09
39	stem per ha ≥ 25cm	300908.4	351089.8	1.166766	16.68
17	stem per ha ≥ 25cm	421200.9	468914.6	1.11328	11.33
17	scale	466.4352	608.0634	1.30364	30.36
34	scale	62.91642	73.88892	1.174398	17.44
39	scale	1300.957	2110.377	1.622173	62.22
34	shape	7.115028	7.862628	1.105073	10.51
39	shape	60.78485	73.45505	1.208443	20.84
41	shape	5.73634	16.92312	2.95016	195.02
45	shape	15.19626	30.32781	1.995742	99.57

6. **Discussion**

This work attempts to investigate the spatial continuity of tree diameter distribution using both parametric and non-parametric approaches at compartment level. For the parametric case, truncated two parameter Weibull distribution function is found better to describe the tree diameter distribution under study and that any effort to overlook truncated data may adversely influence conclusion. In forestry practice, it seems uncommon to use truncated distribution for parameter estimation, however it requires caution when samples are taken from a portion of the population, for instance, when lower diameter sizes are ignored in sampling design. The effect of such practice could be pronouncing in young stands and in natural forests where regeneration is a continuos process.

The shape and scale parameters of the truncated Weibull function are estimated using maximum likelihood method at plot level. Consequently, both parameter estimates are subjected for spatial continuity study as attributes of tree dbh distribution using isotropic semivariogram and corresponding models. In the non-parametric approach, the cumulative tree dbh distribution function (cdf) at a single cutoff for each plot in the compartment is computed and examined for spatial continuity. The spatial continuity study of number of stems per hectare greater or equal to the selected cutoff is also considered. As shown in Figures 2, 3, 6, and 9, most of the variograms of Weibull parameter estimates, cdf and number of stems as attributes of tree dbh distribution seem to exhibit steady spatial continuity. Even though no apparent reason may be given at this point, the older stands seem to exhibit better and consistent spatial continuity in the variables considered. Since variograms depend on lag distance, the reported variograms are based on the lag distance that yields better spatial continuity. Accordingly, the lag distance that result in the reported variograms varies from one compartment to another which in turn may depends on the stand characteristics. As regards to the non parametric approach, even though a single cutoff is considered , it is important to note that the variogram could vary with cutoffs. For estimation purpose, spatial functions of more cutoffs are required.

The parameters for the variogram models considered in this study are estimated using ordinary non-linear (NLS) and weighted least squares (WLS), Cressie (1985) methods. In most cases, particularly, when the variogram shows steady continuity, both methods of parameter estimation show the same performance, as it can be observed from their respective model plots. This observation agrees with the study of Zimmerman and Zimmerman(1991). However, the WLS is used for kriging in the cross validation study. Mcbratney and Webster(1986) have recommended WLS as reliable and computationally efficient.

Despite that in some compartments the sample size or the number of pairs in a lag considered in this study is small as compared to the size known to be "a rule of thumb" (30-50 pairs), the variograms presented for Weibull parameter estimates, cdf, and stems per hectare in most compartments seem to be very indicative to suggest that the tree diameter distribution under study is a regionalized variable. Thus, this exhibited spatial continuity of tree diameter distribution may be modelled and used for estimation purpose. However, when it comes to interpolation, particularly with the parametric approach, the size of plot associated with the number of trees, distribution function that is assumed to completely describe all plots in the compartment, design of sampling, optimal spacing between plots, and effect of the distance of samples from estimation location are prior points of investigation. It is understood that a slight variation in parameter estimates may yield different diameter distribution. This sensitive character of the parameter estimates may not be encouraging for practical application. Another point of importance in practice is distance between prediction location and samples. It may be rarely of practical importance to take samples very closer to the estimation location. The results in this paper shows estimation considered from samples located 25 meters or more distance away from the prediction location is not encouraging. The SSR has increased considerably (see Table 12) and variability seem to increase since in 66.7% of the variables their RMS values exceeded one as compared to 26.7% in the prediction from the cross validation study. Hence, it is recommended to carry out comprehensive study of the problem in order to arrive at conclusive remark.

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Appendix 1: **Sample design description**

In principle, for each compartment plots were arranged in a systematic grid, with randomly chosen starting point. The spacing was chosen to give about ten plots per compartment (a bit more or less depending on size of the compartment). There is one exception: In compartment 43 two grids were laid out, where the second starts 50 m SW from the starting point of the first. In addition to these main plots, short span plots, called satellite plots, were laid out from the main plots in an effort to provide useful data for the variogram function estimation. The direction (N, S, E, W) for the satellite plots were chosen randomly. For the compartments 17 and 39 two satellite plots (at 90^{o} angle) were laid out for each main plot. The spacing between plot centers is shown in the following table.

Spacing in meters

Comp.	Main plot	Satellite -1	Satellite -2
17	80	20	25
34	50	15	-
39	40	12	15
41	40	12	-
43	100	20	-
43	100	20	-
45	50	15	-

Appendix 2: **Lognormal and Weibull parameter estimates for plots and computed chi-squares**

				Lognormal est.			Weibull estimates		
Comp	Plot	Loc	Stems	Mean	St.d	chi-sq	Scale	Shape	chi-sq
17	1	[0,0]	21	2.86420	0.49374	1.28471	21.53088	2.58996	0.40853
17	2	[-80,0]	22	3.02919	0.34619	0.48800	24.21906	3.69599	0.06737
17	3	[-160,0]	23	2.97800	0.17837	0.58793	21.57122	5.34032	0.64000
17	4	[0,80]	22	2.93667	0.49886	2.39628	23.18505	3.00899	0.29461
17	5	[-80,80]	13	3.24988	0.22063	-	28.80564	4.47559	-
17	6	[-160,80]	35	2.75842	0.26626	0.68635	17.82430	4.66601	0.98546
17	7	[-80,160]	27	3.12574	0.26876	0.97878	25.91030	4.15709	1.06410
17	8	[0,160]	30	3.08812	0.30836	0.49771	25.41941	3.60239	0.22520
17	9	[0,240]	21	3.16083	0.27501	0.36072	26.92946	4.09477	0.34885
17	10	[-80,240]	30	2.93450	0.28470	0.35724	21.57461	3.81549	0.13311
17	11	[-80,300]	34	2.68836	0.36388	0.56932	17.20402	3.21539	0.17706
17	12	[0,300]	39	2.45364	0.64017	4.14378	14.22156	1.51881	2.79247
17	13	[30,-63]	16	3.21374	0.32995	0.12805	28.91452	3.99297	0.38015
17	14	[0,100]	24	3.13601	0.30792	0.66350	26.45088	4.27373	0.16102
17	15	[-100,80]	24	3.29276	0.21288	0.61885	29.93876	4.96456	0.96731
17	16	[-160,60]	26	3.01993	0.28119	0.57772	23.53861	3.70145	2.22377
17	17	[-60,160]	19	3.30150	0.28782	1.65171	30.73784	5.07268	0.69229
17	18	[0,180]	22	3.01810	0.33113	0.06359	23.91208	3.47471	0.15028
17	19	[-20,240]	18	3.24952	0.24913	0.08259	29.13016	4.32493	0.35036
17	20	[-80,220]	30	2.97311	0.25933	0.76696	22.13596	4.31967	0.98504
17	21	[-60,300]	41	2.76110	0.29396	1.66433	18.07814	4.07545	0.61381
17	22	[0,280]	19	3.04003	0.46540	1.09447	25.63916	2.45628	1.11444
17	23	[0,20]	22	3.21795	0.29647	2.85340	28.57295	4.10288	1.38931
17	24	[-60,0]	25	3.04583	0.33093	3.66488	23.96015	4.64439	0.31762
17	25	[-160,-20]	33	2.69467	0.24697	0.32867	16.65367	4.49364	0.75316
17	26	[10,-63]	24	2.80528	0.58065	2.04069	20.64671	1.85519	1.19335
17	27	[25,0]	22	2.99008	0.46008	4.16859	24.24647	2.97735	1.73402
17	28	[-80,25]	20	3.07687	0.39313	1.19975	25.59818	3.37726	0.36369
17	29	[-135,0]	30	2.92487	0.26619	6.28455	21.21001	3.64985	11.06591
17	30	[-25,80]	18	3.26845	0.32454	1.31827	30.42386	3.91648	0.80490
17	31	[-80,105]	20	3.30373	0.27669	0.88515	30.87227	4.67751	0.37778
17	32	[-135,80]	21	2.82868	0.56480	0.76248	21.41874	2.46037	0.12355
17	33	[-80,135]	28	3.25029	0.25399	0.68580	29.20154	4.42974	1.14526
17	34	[-25,160]	23	3.19827	0.21775	3.16625	27.01245	5.39810	1.75590
17	35	[0,215]	24	3.06679	0.31457	3.93315	24.54582	4.67306	0.63663
17	36	[-55,240]	23	3.06234	0.37171	3.95790	24.96332	4.04365	0.63228
17	37	[-80,275]	30	3.00792	0.31210	0.82699	23.36678	3.76410	1.79783
17	38	[-25,300]	43	2.65566	0.41633	2.07363	16.73437	2.26978	3.10300
17	39	[30,-88]	18	3.28342	0.32232	0.42917	31.01847	3.80114	0.28792
34	1	[125,150]	34	2.49714	0.43294	1.28860	14.38953	2.50225	1.22502
34	2	[75,150]	41	2.60759	0.31309	4.01333	15.59314	3.71513	2.43566
34	3	[25,150]	40	2.77119	0.29360	0.91005	18.35055	3.62072	0.19733

34	4 [0,150]	35	2.86606	0.30191	2.23525	20.16652	4.18481	0.89359
34	5 [25,200]	39	2.64848	0.27992	1.45083	16.09025	3.64752	1.96135
34	6 [75,200]	44	2.56617	0.28111	0.63689	14.78725	3.74680	2.06621
34	7 [75,100]	35	2.71343	0.38237	1.42043	17.81781	2.88602	0.67122
34	8 [25,100]	35	2.81616	0.30588	0.56576	19.18291	3.96472	0.55971
34	9 [0,100]	42	2.76171	0.32714	2.65932	18.34230	3.21285	1.58005
34	10[0,50]	36	2.54046	0.41527	1.15646	14.91346	2.46318	1.32997
34	11[0,0]	40	2.68169	0.40567	1.13045	17.38613	2.82846	1.35753
34	12[25,50]	51	2.52755	0.39264	2.87751	14.66819	2.72130	1.89798
34	13[125,135]	45	2.32272	0.42677	0.89015	11.38810	2.02785	0.88330
34	14[90,150]	36	2.42909	0.45409	0.44753	13.33969	2.23027	0.42395
34	15[25,135]	48	2.71009	0.26635	3.61745	17.00179	4.30473	3.32982
34	16[0,135]	30	2.84246	0.23559	5.88108	19.11889	5.03028	3.03937
34	17[10,200]	33	2.83631	0.24625	0.76472	19.09934	5.03288	1.51266
34	18[75,215]	41	2.51125	0.34535	3.31853	14.25024	3.36278	4.05011
34	19[60,100]	36	2.72878	0.39941	5.47909	18.23995	2.96985	2.95724
34	20[40,100]	43	2.78244	0.32121	2.16706	18.70480	3.63456	1.91464
34	21[-15,100]	45	2.69621	0.37574	9.49869	17.50160	2.97264	5.97813
34	22[15,50]	43	2.61299	0.41078	2.17630	16.14902	2.62608	0.77530
34	23[0,15]	39	2.56819	0.41438	2.20247	15.43881	2.75712	1.52208
34	24[25,35]	40	2.69896	0.41455	2.28260	17.72830	2.79608	1.29889
39	1 [80,0]	11	3.22047	0.36263	-	29.77103	3.31927	-
39	2 [80,40]	13	3.22364	0.41006	-	29.68474	3.59188	-
39	3 [80,80]	28	2.33131	0.91676	3.85598	14.48497	1.11624	4.66093
39	4 [80,120]	30	0.97981	1.18839	3.68556	2.53913	0.59225	4.32202
39	5 [40,40]	20	3.19667	0.23487	0.06594	27.36275	4.81959	0.25733
39	6 [40,80]	15	3.15003	0.46864	1.46776	27.87221	3.62616	0.10168
39	7 [40,120]	25	2.51481	0.73976	3.26635	16.09215	1.39778	3.71333
39	8 [40,160]	100	1.56753	0.67766	7.06091	3.35750	0.88409	6.50129
39	9 [0,160]	15	2.89396	0.58868	0.09608	22.78338	1.82494	0.11355
39	10[0,200]	45	2.28453	0.46153	8.26720	11.28555	2.05700	6.56360
39	11[0,240]	38	1.68243	0.71847	0.82702	3.58166	0.81996	0.72266
39	12[0,280]	61	2.05649	0.55351	3.20399	7.53673	1.28806	3.21025
39	13[68,0]	13	3.37140	0.17977	-	31.84641	5.96499	-
39	14[80,28]	15	2.58916	0.84412	0.37404	17.19344	1.13661	0.20913
39	15[68,80]	16	3.12003	0.58187	1.07597	28.93607	2.46068	0.24246
39	16[68,120]	37	1.79711	0.90095	3.35024	5.92344	0.82002	3.49501
39	17[52,40]	14	3.32169	0.19774	-	30.23273	6.82267	-
39	18[40,68]	16	3.07285	0.41971	1.03953	25.37646	3.84165	0.38784
39	19[28,120]	17	3.08094	0.47233	0.92440	26.59939	3.03870	0.19734
39	20[40,148]	52	1.82266	0.70262	6.85615	6.09902	1.05375	5.77776
39	21[0,172]	17	2.87869	0.45309	0.37849	21.64282	2.94160	0.53728
39	22[0,212]	27	0.24262	1.29674	0.31239	0.18662	0.36285	0.35751
39	23[-12,240]	25	2.72796	0.50419	0.74553	18.79188	2.23171	0.33363
39	24[0,268]	58	1.64984	0.63588	0.45317	3.87894	0.96152	0.48141
39	25[80,15]	14	3.33300	0.40171	-	33.70716	3.22174	-

39	26[65,40]	9	3.12915	0.47494	-	27.13949	4.25342	-
39	27[80,65]	27	3.17874	0.26346	0.72871	27.00103	5.16214	0.56872
39	28[80,105]	31	2.65800	0.56815	1.25688	17.89777	2.02277	1.49678
39	29[40,25]	17	3.25702	0.15333	0.73250	27.94702	7.30099	1.23844
39	30[55,80]	14	3.44981	0.18469	-	34.39504	6.40136	-
39	31[40,135]	49	2.23190	0.56564	5.27112	10.54439	1.54580	3.74703
39	32[25,160]	42	2.33676	0.59323	3.09258	12.58331	1.68078	2.00303
39	33[-15,160]	22	3.08493	0.40656	0.05765	26.38894	2.77586	0.13193
39	34[15,200]	52	2.09748	0.47844	10.22219	9.41267	2.00287	10.26168
39	35[0,255]	60	0.73363	0.93209	7.54508	1.13172	0.60894	8.20727
39	36[-15,280]	29	2.73474	0.53861	1.37665	19.25364	2.29482	1.45707
41	1 [40,160]	34	2.45217	0.45275	6.95612	13.96142	2.59228	8.94921
41	2 [40,120]	49	2.38881	0.46664	2.84843	12.34666	1.89012	2.41851
41	3 [40,80]	44	2.44143	0.49426	2.89173	13.33431	1.88058	1.92729
41	4 [70,40]	34	2.65891	0.44642	1.45471	17.08563	2.33301	1.44575
41	5 [40,40]	33	2.67620	0.34512	0.47752	16.91764	3.14004	0.39817
41	6 [40,0]	32	2.73866	0.25837	6.07301	17.19157	5.98031	1.86571
41	7 [0,120]	48	2.47459	0.37375	1.14248	13.79959	2.90994	1.31109
41	8 [0,160]	21	2.56719	0.29265	0.15847	14.89645	3.47803	0.13406
41	9 [40,148]	26	2.53021	0.38051	2.50312	14.43799	2.40405	1.37834
41	10[28,120]	50	2.42159	0.36513	1.24827	12.76256	2.47605	2.10111
41	11[28,80]	63	2.03812	0.57936	2.71678	7.68771	1.28925	2.73622
41	12[70,28]	23	2.86691	0.37439	1.35482	20.79196	3.20180	1.76874
41	13[52,40]	28	2.83738	0.31110	2.05832	19.79144	3.63456	1.65123
41	14[40,12]	33	2.72335	0.20344	1.81134	16.77435	5.75609	1.40657
41	15[0,132]	46	2.48231	0.27138	0.69496	13.52763	3.99998	0.68203
41	16[12,160]	24	2.40503	0.40340	0.28576	12.60790	2.25466	0.44194
43	1 [100,500]	39	2.53467	0.30349	0.54050	14.37585	3.57630	1.44318
43	2 [100,400]	46	2.37028	0.45879	1.77544	12.38771	2.09304	1.03293
43	3 [0,400]	46	2.42273	0.37443	2.18406	12.96902	2.65103	1.61132
43	4 [0,300]	90	2.56610	0.32100	1.74960	14.85312	2.75349	6.96741
43	5 [100,100]	44	2.44998	0.35226	0.65632	13.24097	2.69128	1.14923
43	6 [100,0]	42	2.35747	0.41540	4.64230	12.25111	2.44026	3.18615
43	7 [200,0]	42	2.46099	0.48256	9.28267	13.82324	2.04555	8.32862
43	8 [200,100]	43	2.65328	0.31059	1.54714	16.27685	3.68697	1.81156
43	9 [200,200]	34	2.44457	0.35472	1.37449	13.38365	3.39088	2.50311
43	10[100,480]	47	2.58298	0.27436	1.94367	15.00589	3.88909	4.54480
43	11[80,400]	53	2.41885	0.36213	1.10043	12.79940	2.63551	1.51130
43	12[20,400]	51	2.43024	0.34355	4.69396	13.05774	3.06197	2.76090
43	13[0,320]	77	2.55849	0.31353	7.63237	14.81720	3.55878	3.68146
43	14[80,100]	36	2.40139	0.45038	2.34857	12.89422	2.21634	1.38029
43	15[100,-20]	46	2.49152	0.45638	1.89862	14.04781	2.07050	1.98572
43	16[200,20]	44	2.58090	0.39444	3.05766	15.43270	2.50525	4.52833
43	17[180,100]	40	2.56348	0.41644	0.47748	14.99273	2.13531	1.18736
43	18[220,200]	27	2.58659	0.33333	0.94591	15.35263	3.38622	1.46953
43	19[65,465]	57	2.39878	0.41133	4.44247	12.94703	2.71482	2.58265

43	20	[65,365]	46	2.41201	0.39444	3.06223	12.30333	2.04178	4.47737
43	21	[65,65]	49	2.20721	0.65720	5.06727	10.03609	1.25667	5.04525
43	22	[65,-35]	61	2.50453	0.29724	6.22953	13.71476	2.68153	16.23396
43	23	[165,-35]	40	2.33947	0.43153	1.98456	11.73380	2.07158	2.58846
43	24	[165,65]	41	2.75623	0.32674	0.27337	18.30985	3.09873	0.64951
43	25	[165,165]	62	2.46057	0.37940	13.97304	13.44674	2.57610	12.24193
43	26	[165,265]	48	2.60751	0.40488	1.13562	15.97830	2.54322	2.58979
43	27	[165,365]	30	2.75608	0.28917	4.86791	18.05171	3.72379	8.07108
43	28	[245,165]	31	2.47933	0.26251	1.18502	13.39149	4.59814	1.58157
43	29	[85,465]	56	2.62635	0.27732	2.90914	15.63815	4.21100	2.52890
43	30	[65,385]	60	2.45672	0.33667	7.04617	13.33137	2.93740	7.80305
43	31	[65,85]	50	2.34650	0.53508	4.53604	12.53969	1.89713	3.94336
43	32	[85,-35]	48	2.61843	0.38173	4.85262	16.10776	2.89597	5.96033
43	33	[145,-35]	30	2.45051	0.38165	1.18959	13.25016	2.38885	0.99020
43	34	[165,45]	51	2.76338	0.36366	2.29848	18.65093	3.00530	3.79852
43	35	[165,145]	53	2.47885	0.44200	1.10266	14.13447	2.43415	0.85972
43	36	[145,265]	59	2.54604	0.37438	9.69387	14.85873	2.74434	10.29655
43	37	[145,365]	40	2.70404	0.30992	0.67153	17.25097	3.26912	1.89485
43	38	[245,185]	43	2.29164	0.40522	1.13644	11.31527	2.41304	0.61171
45	1	[175,200]	27	2.66711	0.32357	0.16770	16.65389	3.51555	0.48083
45	2	[125,200]	40	2.17997	0.35096	1.63675	9.82454	2.65154	1.18020
45	3	[125,150]	26	2.51618	0.45336	1.64229	15.00353	2.92517	2.36482
45	4	[175,150]	41	2.42223	0.38262	1.48726	13.13013	3.00815	0.77581
45	5	[125,100]	37	2.65123	0.19960	4.00774	15.56130	6.02339	1.56787
45	6	[25,100]	50	2.43569	0.30491	4.37097	13.02635	3.93827	1.58744
45	7	[25,50]	37	2.51126	0.27366	1.11239	13.93573	3.73092	3.11929
45	8	[50,0]	42	2.16821	0.49513	4.59370	9.41709	1.65155	4.18840
45	9	[75,50]	36	2.40690	0.30241	0.36586	12.54249	3.20249	0.23574
45	10	[0,50]	44	2.46456	0.34358	5.55352	13.56726	3.24501	2.40780
45	11	[25,150]	34	2.50843	0.36779	0.57007	14.34931	3.23574	0.29488
45	12	[175,185]	36	2.48715	0.47227	2.06605	14.53599	2.45788	2.39530
45	13	[135,200]	30	2.43777	0.42178	2.53244	13.58104	2.75238	2.77748
45	14	[110,150]	37	2.27109	0.40617	2.45759	10.95596	2.30395	2.05332
45	15	[200,150]	36	2.55086	0.30205	1.45471	14.61575	3.55654	1.53246
45	16	[125,115]	29	2.57751	0.25925	0.34701	14.81346	4.26755	0.93835
45	17	[40,100]	40	2.42215	0.25391	1.83887	12.62338	4.25989	1.17211
45	18	[25,65]	34	2.46479	0.34154	2.93850	13.54449	3.51757	0.86043
45	19	[10,0]	30	1.99270	0.55086	0.78520	6.06740	1.13048	1.00516
45	20	[75,65]	46	2.20316	0.34565	3.88679	10.06466	2.67976	3.78519
45	21	[15,50]	47	2.48716	0.27731	4.67068	13.56613	4.61257	2.24868
45	22	[10,150]	35	2.49122	0.36526	1.27906	14.06660	3.18031	1.74729

Appendix 3: **Stem and leaf plot of the SPR from cross validation**
(Leaf unit = 0.01), computed in MINITAB

Compartment 17
cdf, cutoff =25 cm

```
  2   -1 65
  9   -1 4321110
 15   -0 966655
 18   -0 431
 (5)   0 02234
 16    0 5566677999
  6    1 00144
  1    1 8
```

Compartment 39
Cdf, cutoff = 25 cm

```
  3   -1 977
  5   -1 10
 11   -0 887755
 17   -0 433310
(11)   0 11222333444
  8    0 5699
  4    1 03
  2    1
  2    2 0
  1    2 6
```

Compartment 43
Cdf, cutoff =18cm

```
  2   -2 43
  4   -1 96
  6   -1 20
 10   -0 7775
 15   -0 44320
(12)   0 000122233444
 11    0 557789
  5    1 01
  3    1 77
  1    2 0
```

Compartment 45
Cdf,cutoff = 18cm

```
  1   -2 1
  3   -1 65
  4   -1 4
  6   -0 75
  8   -0 11
 (6)   0 000113
  8    0 57779
  3    1 00
  1    1 8
```

Compartment 45
Stem per/ha ≥ 18cm

```
  1   -1 7
  4   -1 111
  6   -0 77
 (7)  -0 3311000
  9    0 0124
  5    0 89
  3    1 4
  2    1 6
  1    2 0
```

Compartment 43
Stem per/ha ≥ 18cm

```
  2   -1 75
  5   -1 310
 15   -0 9988777666
 (8)  -0 33222210
 15    0 4444
 11    0 5699
  7    1 12344
  2    1
  2    2 13
```

Compartment 39
Stem per/ha ≥ 25cm

```
  1   -2 5
  1   -2
  1   -1
  3   -1 10
 14   -0 99888776655
 18   -0 4222
 18    0 11223344
 10    0 579
  7    1 00122
  2    1 6
  1    2 1
```

Compartment 17
Stem per/ha ≥ 25cm

```
  2   -1 96
  8   -1 433000
 14   -0 977776
 17   -0 420
 (9)   0 001223344
 13    0 566778
  7    1 01123
  2    1 69
```

Compartment 17
Scale parameter estimate

1	-2 1
3	-1 85
6	-1 322
11	-0 97766
(9)	-0 443332110
19	0 0012344
12	0 6789
8	1 011114
2	1 57

Compartment 34
Scale parameter estimate

2	-2 32
5	-1 755
6	-1 1
9	-0 755
10	-0 1
(5)	0 01223
9	0
9	1 0001223
2	1 69

Compartment 39
Scale parameter estimate

2	-1 96
5	-1 310
10	-0 87765
18	-0 44322100
18	0 00223344
10	0 56689
5	1 012
2	1 59

Compartment 34
Shape parameter estimate

3	-1 875
4	-1 3
9	-0 97665
12	-0 300
12	0 1222
8	0 589
5	1 1123
1	1
1	2 3

Compartment 39
Shape parameter estimate

1	-2 0
1	-1
3	-1 40
9	-0 765555
(12)	-0 444433222210
15	0 0122224
8	0 667
5	1 4
4	1 567
1	2 3

Compartment 41
Shape parameter estimate

1	-1 8
4	-1 200
6	-0 97
8	-0 20
8	0 1
7	0 56899
2	1 2
1	1 6

Compartment 45
Shape parameter estimate

1	-1 7
3	-1 40
7	-0 9875
(5)	-0 43100
10	0 00114
5	0 679
2	1
2	1 8
1	2
1	2 6

Appendix 4: **Stem and leaf plot for the SPR from distance restricted samples.**
(Leaf unit = 0.01)

Compartment 17
cdf, cutoff = 25cm

```
1    -2 3
3    -1 65
6    -1 330
13   -0 9996555
17   -0 3322
19    0 22
(14)  0 55666677788999
6     1 0123
2     1 67
```

Compartment 39
Cdf, cutoff = 25cm

```
1    -2 1
2    -1 5
6    -1 4310
12   -0 987665
16   -0 4332
(5)   0 00144
15    0 5555677888999
2     1 0
1     1 9
```

Compartment 43
Cdf, cutoff =18cm

```
2    -2 42
5    -1 766
7    -1 21
12   -0 99755
15   -0 432
(12)  0 001222234444
11    0 556667
5     1 14
3     1 9
2     2 00
```

Compartment 45
cdf, cutoff =18cm

```
3    -2 310
4    -1 7
4    -1
7    -0 987
9    -0 00
(5)   0 33444
8     0 678
5     1 0123
1     1 9
```

Compartment 17
Scale parameter estimate

```
1    -2 1
4    -1 665
8    -1 3200
14   -0 776666
(7)  -0 4432100
18    0 001233
12    0 788899
6     1 0123
2     1 6
1     2 3
```

Compartment 34
Scale parameter estimate

```
1    -2 5
2    -2 2
4    -1 66
7    -1 311
8    -0 5
9    -0 4
(8)   0 11223444
7     0 5
6     1 44
4     1 579
1     2 0
```

Compartment 39
Scale parameter estimate

```
2    -1 76
8    -1 331100
11   -0 855
(8)  -0 44333211
17    0 012224
11    0 8
10    1 000011234
1     1 5
```

Compartment 34
Shape parameter estimate

```
1    -2 0
2    -1 5
5    -1 411
9    -0 9655
11   -0 40
(5)   0 00133
8     0 799
5     1 0123
1     1
1     2 4
```

Compartment 39
Shape parameter estimate

1	-2	3
1	-1	
1	-1	
11	-0	9887766555
(12)	-0	443333211000
13	0	12234
8	0	556
5	1	0
4	1	579
1	2	0

Compartment 41
Shape parameter estimate

2	-1	77
3	-1	4
7	-0	7655
7	-0	
(3)	0	122
6	0	578
3	1	3
2	1	58

Compartment 45
Shape parameter estimate

1	-2	2
2	-1	5
4	-1	41
8	-0	5555
(6)	-0	111000
8	0	04
6	0	559
3	1	
3	1	56
1	2	
1	2	
1	3	0

Compartment 17
Stem per ha ≥ 25cm

1	-2	2
3	-1	66
7	-1	3210
14	-0	9987655
(7)	-0	1111000
18	0	001244
12	0	66678
7	1	0001
3	1	567

Compartment 39
Stem per ha ≥ 25cm

1	-2	4
1	-1	
5	-1	1100
15	-0	9998776655
17	-0	43
(6)	0	111244
13	0	55788
8	1	011233
2	1	68

Compartment 43
Stem per ha ≥ 18cm

3	-1	866
5	-1	40
15	-0	9766666555
19	-0	4432
19	0	0001233
12	0	56789
7	1	233
4	1	67
2	2	0
1	2	9

Compartment 45
Stem per ha ≥ 18cm

2	-1	95
5	-1	211
8	-0	875
(6)	-0	422111
8	0	1
7	0	69
5	1	03
3	1	78
1	2	
1	2	5