Historical substitution between renewable and fossil Energy

With historical data from Sweden during 19th and 20th century

Evert Jonsson

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Supervisor:	**Robert Hart, Swedish University of Agricultural Sciences, Department of Economics**
Assistant supervisor:	Pedro Naso, Swedish University of Agricultural Sciences, Department of Economics
Examiner:	Shon Ferguson, Swedish University of Agricultural Sciences, Department of Economics

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Abstract

This study examines Sweden's long-term substitution between renewable and fossil energy during the 19th and 20th centuries. I estimate the elasticity of substitution between woodfuel and coal energy to quantify how easily Swedish users switched fuels in response to changes in relative prices or availability. Using annual historical data, the project employs a Constant Elasticity of Substitution (CES) production function for the combined energy input, adjusted appropriately for Ordinary Least Squares (OLS) estimation to test the theory. Short-term substitution is measured by detrending the data and re-estimated using OLS. My results show a high long-term elasticity of 4.027, while short-term elasticity is lower with a value of 1.564. These findings indicate that between 1800 and 2000, Swedish consumers could readily substitute coal for woodfuel as relative prices and supply conditions changed, whereas the short-term elasticity showed a smaller substitution effect. The high elasticity points to a historically flexible energy system, allowing consumers to shift consumption patterns effectively.

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1. Introduction

1.1 Background

The transition from fossil fuels to renewable energy is a central challenge for climate policy and has been historically significant for economic development. Historical trends in Sweden illustrate how various policies and decisions significantly impacted both the economy and society. Traditionally, Sweden relied heavily on woodfuel until global trade expanded. Increased trade and improved transportation led to greater usage of coal which is a fossil fuel that is significantly more energy intensive. Easier trade made coal more affordable, encouraging substitution away from renewable energy sources. In this thesis, I examine and discuss the historical elasticity of substitution in Sweden.

Energy consumption in Sweden has historically depended on traditional renewable biomass, made from woodfuel. Kander (2002) provides comprehensive empirical research on Swedish energy consumption from the year 1800 to 2000. Her work reveals that woodfuel was Sweden's primary energy source until the early 20th century, after which coal and other fossil fuels gained prominence due to global trade. Kander's detailed data on historical energy trends offers a valuable foundation for examining elasticity of substitution. This study is complemented by data from Schön (1988), which includes detailed estimates of industrial energy use. Schön's research quantifies total energy consumption in Swedish manufacturing and highlights when and how coal began replacing woodfuel. The shift was gradual and varied across different sectors.

The substitution between woodfuel and coal in Sweden is historically important and contributes to our understanding of contemporary sustainability challenges. Stern and Kander (2012) highlight that when energy resources are inelastic or scarce, economic growth may become constrained. Historically, renewable energy sources like woodfuel were limited by land availability that potentially hindering economic growth until the introduction and increased accessibility of coal, the highly energy-intensive fossil fuel, alleviated these constraints. Rising demand led to a long-term shift toward fossil fuel due to coal's greater availability and relatively lower cost compared to woodfuel, clearly demonstrated by Kander's (2002) long-term data.

Understanding the elasticity of substitution between renewable and fossil energy helps assess how flexibly Sweden can respond to changes in relative prices or technology by switching fuels. This thesis employs a Constant Elasticity of Substitution (CES) production function to model a combined energy input from two sources. The CES function is flexible and lets the elasticity of substitution be estimated rather than assumed, a concept originates with Arrow et al. (1961), who introduced the functional form and showed its usefulness for estimating substitution elasticities.

The optimal mix of inputs can be derived using first-order conditions (FOCs), which express how a firm maximizes profit while producing a fixed level of output (Herzing, n.d), particularly when the production function allows for constant elasticity of substitution. To estimate this elasticity, I logatithmise the CES equation, following Kmenta (1967), who demonstrated how a CES function can be linearized. Running an Ordinary Least Squares (OLS) regression on these transformed FOCs yields a simple estimate of the substitution elasticity.

1.2 Literature Review

The study by Papageorgiou et al. (2013) provides an empirical estimate of the elasticity of substitution between clean and dirty energy inputs at a macroeconomic level. The authors employ a nested CES production function with sectoral panel data from 26 countries to qualify how readily renewable energy can substitute for fossil energy. They find a substitution elasticity significantly greater than one, implying that clean and dirty energy are relatively easy to substitute, especially in the long run. If Sweden's historical energy transition exhibit similarly high elasticity of substitution, this would suggest that policy and technology changes enabled a smooth shift from renewable to fossil energy.

Stern (2010) conducts a meta-analysis of interfuel substitution that synthesizes results from 47 studies on how different energy sources can replace one another across sectors. The paper examines "shadow" elasticities of substitution between coal, gas and electricity in various contexts. It finds that, at industrial-sector level, most fuels are fairly substitutable, with elasticities typically greater than one. At the aggregate economy-wide level, the substitution is more constrained and it appears harder to swap fuels than within a specific industry. These findings inform this thesis by providing benchmarks for typical substitution elasticities and by highlighting the importance of sectoral heterogeneity in Sweden's historical energy transition.

A recent study by Schwerin (2025) examines the long-term elasticity of substitution between fossil fuels and renewable energy sources on a global scale for the period 1800-2012. The research quantifies how easily economies have historically substituted between these two energy types, thereby demonstrating their flexibility. Schwerin reports a central elasticity estimate of 4.33, indicating a high degree of substitutability between fossil and renewable energy over the long run. The paper employs an orthogonal regression approach, within a generalized method of moments (GMM) framework to analyse the relationship between relative prices and quantities of fossil and renewable energy. These results are directly relevant to this thesis, which investigates the elasticity of substitution between woodfuel and coal in Sweden using long-term data. The high estimate underscores the potential for significant energy transitions over time.

This thesis fills a gap in the literature by providing an estimate of the long-term constant elasticity of substitution between renewable and fossil energy sources in Sweden between the years 1800-2000. It also estimates short-term elasticity by detrending the long-term series, thereby extending existing research on Sweden's historical energy substitution.

1.3 Study overview

This thesis draws on annual data compiled by Kander (2002) covering the period 1800-2000, which report total energy consumption and prices for each source. A constant Elasticity of Substitution (CES) production function is embedded in a profit-maximisation framework. After deriving the first-order conditions and applying a logarithmic transformation, the theoretical model is tested in the econometric model that is Ordinary least squares (OLS). The OLS regression yields an estimated long-run elasticity of 4.027, indicating substantial substitutability between renewable and fossil energy. To measure short-term substitution, the data are detrended and re-estimated which shows an elasticity of 1.564, suggests that substitution remains possible but is more limited from year to year.

This thesis starts to describe the data that are used and how the data is calculated and created. It will then explain the mythology and which steps that are made to make it possible to test the theory with an OLS regression. In the methodology there will be a section that includes and describes the econometrical models. Later, the result will be presented, followed by a detailed discussion and interpretation of the findings. This thesis will end in a conclusion.

2. Data description

The empirical analysis draws on several complementary data sources.

2.1.1 Woodfuel

The woodfuel consumption estimates come primarily from Kander (2002) and consist of different methods to measure the consumption and price during the period 1800-2000. Due to lack of data during the 19th century, estimates are based on a national generalization of a 1920-21 värmland country inquiry that includes 666 farm households. The inquiry concluded a consumption of 3.65m3 per head in the northern part of Värmland. In the middle, the consumption is 3.19m3 and in the south the consumption is 2.83m3. With help of these results, Astrid Kander (2002), used a back-casting approach in her thesis. The technique is a model that works backwards to model the developments from a specific benchmark which in this case was 1924. In this estimation, Kander provides reasoning about several factors that could affect the consumption in different ways. First of all, Kander discusses how an increased consumption of woodfuel gradually became more common because of more heated rooms as a result of economic improvement. In the early 19ths it was common that only the kitchen had a stove but it became later common that households had more than one stove. Kander also argues that under the 19th century, the insulation became better which resulted in a reduced consumption of woodfire. The third factor Kander argues about is the technical development in the stoves that made it possible to substitute wood to coal. The stoves became also more efficient and did not need as much wood as before.

2.1.2 Coal

Under the late part of the 19th century woodfuel became partly replaced by coal, especially among the urban households and in the southern part of Sweden because of the insufficiency of wood. The considerable use of coal started in the 1820s with imported coal and the use increased substantially after the 1850s. In the beginning the use were fairly directed towards steam-ships and under 1850s the use become more frequently in railways. In the same time era the use of coal became more frequently in agricultural loco-mobiles and industrial steam machines. In 1840 the coal became a raw material for town-gas which produced gas for outdoor and factory light. Coal quality played a substantial role in the consumption in Sweden because Sweden had only a small coal deposit of low quality themselves and it were only active during the years 1800-1850. Due to the lack of Swedish own coal production, Sweden had to import large quantities and about 90% of Swedish coal imports came from Britain, Britain produced more than 80% of the coal worldwide in the year 1800 and had still more than 50% of the worldwide production in the year 1850. Kander (2002) estimates the coal use in different methods to try and get the data as near to reality as possible. The coal that was used in the industry is roughly estimated by the number of installed steam horsepower in the 1850s and Kander has counted and estimated how much the engines were used and for how many days. This concludes to a total of 6000 HP used and that is later converted to 735 TJ. The total coal consumption is concluded from the transport sector, the industry sector, the agricultural sector and the household's services sector and this data are estimates from Schön, L (1988) data that Kander uses in her project. The total energy sums up to 2.4 PJ in the 1850s and 13.1 PJ in the 1870s.

2.2 Descriptive statistics

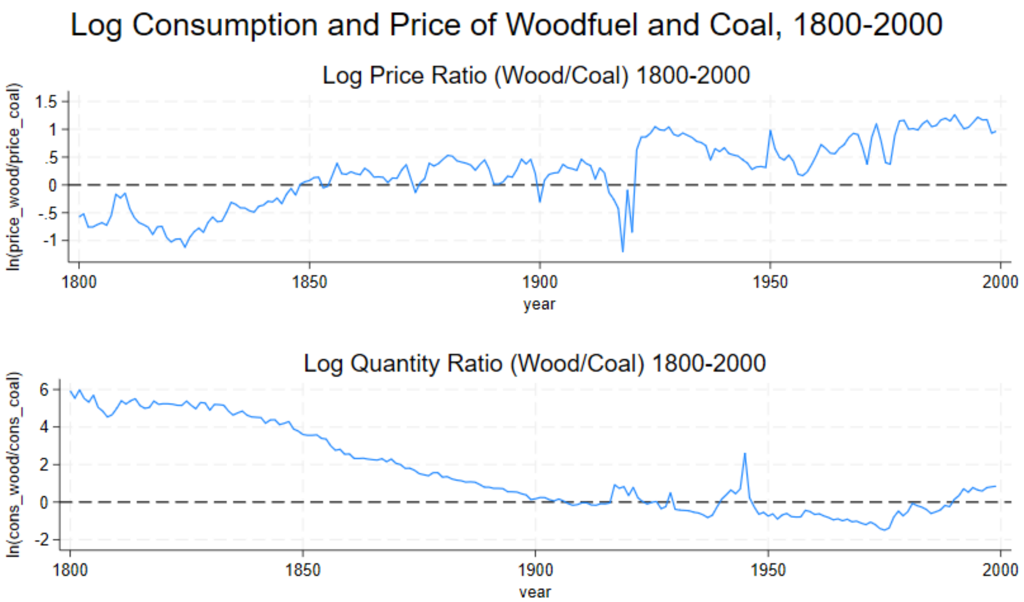

Figure 1, Logaritmized Price and consumption 1800-2000

The two panels in figure 1 plot log ratios, so the horizontal dashed line at 0 marks the point where the two fuels have equal price or quantities. A value of +1 implies that the woodfuel variable exceeds the coal variable by $e^1 \cong 2.7$ times. A value of -1 implies the opposite, that coal is about 2.7 times larger.

At the start of the 19th century, the log price ratio sits just below zero, indicating that wood was slightly cheaper than coal. As transportation cost fell around 1850, coal prices become more competitive and the ratio hovered near zero until World War I. During the war, disrupted transport and tight supplies drove coal prices sharply higher.

In 1800 the log consumption ratio stands a little above +5.0, meaning woodfuel was consumed roughly ninety times more than coal. Over the 19th century the ratio falls steeply as coal use accelerates. Temporary price spikes for coal during World War I and World War II briefly shift consumption back toward woodfuel.

Table 1, Descriptive statistics

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
Year	200	1899	57.88	1800	1999
Consumption_Wood (PJ)	200	104.87	44.60	19.08	326.34
Price_Wood (MJ)	200	11,53	27.00	0,14	119.00
Consumption_Coal (PJ)	200	70.72	68.62	0.22	276.30
Price_Coal (MJ)	200	4.56	8.97	0,285	47.10
Relative quantity wood/coal	200	43.14	78.05	0.22	396.49
Relative price wood/coal	200	1.48	0.81	0.30	3.53
Relative quantity wood/coal (detrend)	200	0.00	0.97	-1.46	2.74
Relative price wood/coal (detrend)	200	-0.00	0.32	-1.59	0.59

Table 1 presents descriptive statistics for the key variables in this study, based on annual observations from 1800-1999. The year 2000 is excluded due to data limitations and price variables have been converted to SEK per MJ for better interpretability. Consumption_wood refers to the annual energy consumption from woodfuel, measured in petajoules (PJ). The average annual consumption of woodfuel is approximately 104.87 and are measured in petajoules (PJ). Price_wood indicate the real annual price of woodfuel, expressed in Swedish kronor (SEK) per megajoule (MJ). Consumption_coal represents annual energy use from coal, also measured in PJ, with a mean of 70.72 PJ. Price_coal is the real unit price of coal in SEK per MJ.

Relative quantity wood/coal is a unit-free ratio calculated by dividing woodfuel consumption by coal consumption for each year. A value above 1 indicates a greater use of woodfuel relative to coal. Relative price wood/coal is the ratio of the price of woodfuel to the price of coal given a year. A value above 1 indicates that woodfuel was more expensive than coal in the given year. Relative quantity wood/coal (detrended) and relative price wood/coal (detrended), represent the year-to-year deviations from the long-term trend in each series. These are used to isolate short-term fluctuations by removing systematic long-term changes in trends.

3. Methodology

3.1 Theoretical framework

In this project, I model the energy sector as a collection of firms that produce an intermediate energy commodity using wood and coal as inputs. The energy producing firms are price-takers in both their input markets and the market for the final output which means that the firms operate under perfect competition. The prices of wood and coal are treated as exogenous which means that no individual firm can influence the price on the market. Given the fixed market prices, each firm chooses how much wood and coal to use in order to maximise profit from energy production.

To capture how firms can substitute between wood and coal the model assumes a constant elasticity of substitution (CES) production function for energy output. This functional form allows firms to adjust their input mix when the relative price of one fuel changes. Profit maximization under these conditions leads each firm to choose an input combination that equalises the marginal product per unit cost of wood and coal. This theoretical setup provides a clear framework for analysing how changes in relative prices can shift demand from renewable to fossil energy.

I apply the same equations to estimate both long-term and short-term elasticity of substitution. The long-term elasticity of substitution reflects how energy consumers respond to gradual price changes over many years, when they have time to adapt technologies and price changes. The short-term elasticity captures responses to temporary price changes, holding structural factors constant and therefore shows how various constraints limit the ability to switch fuels quickly.

3.1.1 Constant elasticity of substitution (CES)

Efficient allocation of energy resources is fundamental to economic productivity and the transition to sustainable energy systems. This section outlines the theoretical framework based on profit maximisation under a Constant Elasticity of Substitution (CES) production function, showing how renewable and fossil energy are optimally allocated. The analysis focuses on woodfuel and coal for the time period 1800-2000. Evaluating how readily consumers substitute between them.

The energy use is assumed fixed, with total consumption denoted by E. This total comprises woodfuel and coal consumption, represented as $E_1 + E_2$, where E_1 and E_2 denote energy from renewable and fossil sources, respectively. Each source has a productivity parameter, A_1 and A_2, capturing how efficiently energy is converted into output. In this project the two productivity terms will be treated as a constant. Output is aggregated with the CES production function introduced by Arrow et al. (1961). The key parameter, η, is the elasticity of substitution between renewable and fossil energy. A low value indicates limited substitutability, whereas a high value means the two sources can be exchanged more easily.

To reach the optimal allocation it's important to assume that the representative firm maximises profit. Profit is defined as total revenue from energy output minus the cost of the two inputs. Revenue equals $w\varepsilon \cdot E$, where E is the total energy output produced with both sources and $w\varepsilon$ is the marginal revenue product of energy, the price per unit of energy output. Costs are w_1E_1 for renewable energy and w_2E_2 for fossil energy, where E_1 and E_2 are the quantities used and w_1 and w_2 are their respective unit prices. Together these components from the profit function (equation 2), which shows how the representative firm chooses its input mix to maximise profit while accounting for the productivity and cost of each energy sources. Changes in relative prices or in productivity parameters determine the degree of substitution between the two fuels.

Equation 1, Standard CES function, Arrow (1961)

$$\mathrm{E} = \left((\mathrm{A}_1 E_1)^{\frac{\eta-1}{\eta}} + (\mathrm{A}_2\mathrm{E}_2)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{1}$$

In this equation, E is the total energy output produced from combining E_1 and E_2. The variable E_1 denotes the quantity of woodfuel, while E_2 denotes the quantity of coal. A_1 and A_2 are efficiency parameter that capture the productivity of woodfuel coal, respectively. η is the elasticity of substitution parameter and indicates how easily woodfuel can be substituted for coal.

Equation 2, Profit function

$$\pi = \mathrm{w_E} \cdot \mathrm{E} - (\mathrm{w_1E_1} + \mathrm{w_2E_2}) \tag{2}$$

I assume a representative firm that maximizes profit and uses two energy sources, as described in equation 2. Here, $\mathrm{w_E}$ is the value of the total energy sold, and E is the total quantity of energy produced. $\mathrm{w_1}$denotes the price of woodfuel, with $\mathrm{E_1}$the quantity of woodfuel used, while $\mathrm{w_2}$ denotes the price of coal, with $\mathrm{E_2}$ represent the quantity of coal produced.

3.1.2 First-order conditions (FOCs)

The CES function from Arrow et al., (1961) is substituted into the profit function. In the context of economic optimisation, First-Order Conditions (FOCs) describe how a representative firm maximises profit while producing a fixed level of output (Herzing, n.d.). Within the CES framework, FOCs determine the optimal mix of inputs when the production function has a constant elasticity of substitution. In this project, the first-order conditions are applied, following Simon and Blume (1994), to allocate a limited amount of energy optimally. I differentiate the profit function with respect to E_1 and E_2, set the derivatives equal to zero, and then divide the two expressions. The resulting condition shows that the optimal energy allocation depends on relative quantities and relative prices, with the elasticity of substitution governing the response.

Equation 3, Profit function with CES substituted

$$\pi_E = w_E((A_1E_1)^{\frac{\eta-1}{\eta}} + (A_2E_2)^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}} - (w_1E_1 + w_2E_2) \tag{3}$$

Here is the profit function expressed with the CES function instead of E. This makes it possible to apply the FOCs and take derivatives with respect to E_1 and E_2.

Equation 4-5, FOCs

$$\frac{\partial \pi_E}{\partial E_1} = 0 \Rightarrow w_1E_1 = w_E \cdot E^{\frac{1}{\eta}} (A_1E_1)^{\frac{\eta-1}{\eta}} \tag{4}$$

$$\frac{\partial \pi_E}{\partial E_2} = 0 \Rightarrow w_2E_2 = w_E \cdot E^{\frac{1}{\eta}} (A_2E_2)^{\frac{\eta-1}{\eta}} \tag{5}$$

With this setup, I take the derivates of the profit function with respect to E_1 and E_2. The resulting FOCs, after rearranging, yield a simple expression where w_1E_1 denotes the price and quantity of woodfuel, while w_2E_2 denotes the price and quantity of coal.

Equation 6-10, Derive the formulae and rearranging the terms:

$$\frac{w_1E_1}{w_2E_2} = (\frac{A_1E_1}{A_2E_2})^{\frac{\eta-1}{\eta}} \tag{6}$$

$$(\frac{w_1E_1}{w_2E_2})^{1-\frac{\eta-1}{\eta}} = (\frac{A_1E_1}{A_2E_2})^{\frac{\eta-1}{\eta}} (\frac{w_1E_1}{w_2E_2})^{-\frac{\eta-1}{\eta}} \tag{7}$$

$$(\frac{w_1E_1}{w_2E_2})^{\frac{1}{\eta}} = (\frac{A_1}{A_2})^{\frac{\eta-1}{\eta}} \cdot (\frac{w_1}{w_2})^{-\frac{\eta-1}{\eta}} \tag{8}$$

$$\left(\frac{w_1E_1}{w_2E_2}\right) = (\frac{A_1}{A_2})^{\eta-1} \cdot (\frac{w_1}{w_2})^{\eta-1} \tag{9}$$

Converting w_1E_1 and w_2E_2 to S_1 and S_2

$$S_i = w_i E_i \Rightarrow \frac{w_1 E_1}{w_2 E_2} = \frac{S_1}{S_2} \quad (10)$$

After rearranging and simplifying, the price-quantity term for woodfuel becomes S_1, while the corresponding term for coal becomes S_2.

Equation 11, Logaritmized function and final formulae

$$\log\left(\frac{S_1}{S_2}\right) = (\eta - 1)\left(\log\frac{A_1}{A_2}\right) - (\eta - 1)\log\left(\frac{w_1}{w_2}\right) \quad (11)$$

Here is the final equation, derivate with FOCs and then logaritmised. Taking logarithms make it possible to see how percent changes in relative price affects the percent change in relative quantity. With this equation, it is possible to estimate the value of η. Unfortunately, lack of data over the technical development in woodfuel (A_1) and coal (A_2) making it hard to track efficiency improvements over the 200-year period. I therefore treat both A_1 and A_2 as constants, together with $(\eta - 1)$ is denoted by α. $-(\eta - 1)$ is denoted by β, when testing the theoretical method with OLS regression.

After deriving the expression, the final expression is a log-linear equation in which the dependent variable is the logarithm of the relative energy ratio and the independent variable is the logarithm of the relative price ratio. Because this form is linear in its parameters, it can be estimated with standard linear-regression methods such as ordinary least squares (OLS). The logarithmic specification also allows clear percentage interpretations and because of that, it is possible to look at the coefficient β that is the key coefficient that will reflect how much input mix changes in response to change in relative price. β will indicate the percent change in the energy input ratio associated with a percent change in the price ratio. When the equation is logarithmised the constant of $A_1$and A_2 will take the symbol α wich is a constant that are not going to be considered anymore due to missing data.

Table 2, Interpret elasticity

$\eta = 0 \rightarrow$	Perfect complements, Inputs cannot be substituted.
$0 < \eta < 1 \rightarrow$	Substitution is possible, but a substantial change in relative prices is required to alter the input mix appreciably.
$\eta = 1 \rightarrow$	Cobb-Douglas case, Inputs can be exchanged at a constant proportional rate while maintaining output.
$\eta > 1 \rightarrow$	High substitutability, Inputs are easy interchangeable which means that a small price difference lead to significant substitution.
$\eta = \infty \rightarrow$	Perfect substitutes, Inputs are fully interchangeable without loss in output.

3.2 Econometric model

3.2.1 Ordinary least squares (OLS)

Since this project uses a CES model that has been reduced to its final form by omitting the technology parameters, I work with the equation corresponding to equation 12. An OLS regression is implemented to test the theoretical relationship and the regression results are used to estimate the elasticity between the two inputs. After transforming the CES first-order condition into the log-linear form described above, OLS estimates the coefficient β by minimizing the sum of squared residuals, producing the best-fit line through the data points, assuming the model is unbiased. To finally estimate the elasticity of substitution, η is derived and interpreted with help from β which will tell the long-term elasticity.

OLS regression is first applied to the long-term dataset to estimate elasticity over a 200-year period, as described in equation 12 and 13. In equation 14 and 15, the long-term data are detrended which makes it possible to see year-to-year fluctuation and a short-term elasticity can be estimated. The future explanation of detrending could be find on page 18.

Equation 12, Formulated to suit an OLS regression

$$\log\left(\frac{S_1}{S_2}\right) = \alpha + \beta \log\left(\frac{w_1}{w_2}\right) \tag{12}$$

Here is the equation, rearranged so it can be estimated with OLS. From this specification I obtain β, which is then used to calculate the elasticity of substitution, η. The term α is a constant that captures the factors omitted from the model.

Equation 13 , Estimate η

$$\beta = -(\eta - 1) \Rightarrow \eta = 1 - \beta \tag{13}$$

This final step yields the value of η. Running the OLS regression provides an estimate of β, which makes it possible to estimate the elasticity.

3.2.2 Detrending

It is possible to estimate the short-term elasticity of substitution between renewable and fossil energy sources by detrending the time series data. Detrending removes persistent long-term trends, such as structural changes and sustained growth. By detrending the data, it becomes possible to isolate the residual ups and downs that earlier were obscured by the overall direction, leaving only the short-term fluctuations. Once these trends are stripped away, movements in relative prices and energy consumption within a given year becomes visible. It is then possible to estimate how the fluctuations around the trend relates to each other. These fluctuations are year-to-year changes in the data and will tell the short-term elasticity.

In this thesis, detrending the series of relative energy price and consumption quantities isolates irregular components of the data, which better reflect short-term behavioural adjustments by energy consumers. These residual deviations represent how energy consumers responded to temporary shocks between woodfuel and coal which will determinate the short-term elasticity.

I first remove long-term trends from the original woodfuel and coal quantities, creating Q_{Wdt} and Q_{Cdt}, and similarly detrend the prices, producing P_{Wdt} and P_{Cdt}. These detrended variables are combined into S_{1dt} and S_{2dt}, The new value of S_{1dt} and S_{2dt} is then replacing the previous into the Equation 14. This is the final expression for the theoretical method which is then simplified like equation 15 to be suitable for OLS regression, which is the econometric method to test the theoretical method. The OLS regression estimate a new value of β_{dt} which makes it possible to estimate the short-term elasticity of substitution with help of equation 9.

Equation 14, Logaritmized function and final formulae

$$\log\left(\frac{S_{1dt}}{S_{2dt}}\right) = (\eta_{dt} - 1)\left(\log\frac{A_{1dt}}{A_{2dt}}\right) - (\eta_{dt} - 1)\log\left(\frac{W_{1dt}}{W_{2dt}}\right) \qquad (14)$$

Same equation as the long-term but with variables that is detrended.

Equation 15, Formulated to suit an OLS regression

$$\log\left(\frac{S_{1dt}}{S_{2}dt}\right) = \alpha_{dt} + \beta_{dt}\log\left(\frac{W_{1dt}}{W_{2}dt}\right) \qquad (15)$$

Same equation as the long-term but with variables that is detrended.

Equation 16 , Estimate η_{dt}

$$\beta_{dt} = -(\eta_{dt} - 1) \Rightarrow \eta_{dt} = 1 - \beta_{dt} \qquad (16)$$

This final step yields the value of η_{dt}. By running OLS, I get the value of β_{dt} which makes it possible to estimate the short-term elasticity.

For β and β_{dt} to serve as unbiased estimates of short- and long-term substitution elasticities, several conditions must be hold. Short-term movements in the woodfuel-to-coal price ratio must be exogenous which means it is uncorrelated with unobserved shocks in the error term. Although detrending removes long-term technological drift, it does not guard against short-term disturbances such as abrupt tariff changes or macroeconomic contractions that can move both prices and quantities. After detrending, the residual price and quantity series must be covariance-stationarity, otherwise, standard OLS inference is invalid and β_{dt} will not reliably capture the contemporaneous link between price and quantity deviations. Any measurement error in price and quantities must also be classical, random rather than systematic. Because these assumptions are difficult to guarantee, it is prudent to add control variables like weather or GDP-growth dummies and to conduct robustness checks with alternative specifications to reduce the threat of omitted-variable bias.

4. Results

Table 3, Elasticity of substitution, long-and short term between the years 1800 and 2000

1800-2000	β
Long-term	-3,027
Short-term	-0,564

A table that shows the result of β in the long- term and short-term.

Table 4, Result long-term

Long-term	(1) ln(Qw/Qc)
ln(Pw/Pc)	-3.027***
	(0.158)
Constant	2.277***
	(0.101)
Observations	200
R-squared	0.650

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The coefficient of -3.027 implies that a 1% rise in relative price of woodfuel is associated with a 3.027 % fall in the relative quantity ratio over 200 years. $\ln(\frac{Qw}{Qc})$ shows the dependent variable. $\ln(\frac{Pw}{Pc})$ shows the main regressor. The high R^2 value indicates that the simple CES specification captures most of the slow-moving variation in input shares.

Table 5, Result short-term

Short-term	(2) ln(Qw_d/Qc_d)
ln(Pw_d/Pc_d)	-0.564***
	(0.216)
Constant	0.000
	(0.068)
Observations	200
R-squared	0.033

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

After detrending, the price-shares link weakens. β_{dt} is -0564, which is still significant at 1%. ln($\frac{Qw}{Qc}$) shows the dependent variable. ln($\frac{Pw}{Pc}$) shows the main regressor. The low R^2 is expected, because year-to-year fluctuations are dominated by idiosyncratic shocks and measurement noise once the common trend has been removed.

A value of -3.027, with a std. err. of 0.158 indicates strong substitutability between woodfuel and coal over the 1800-2000 but it has to be applicated in to the final formula to see the exact value. The value of β is statistically significant at 1% level. The 95 % conf. interval is -3.334 - -2.716.

The β coefficient from this regression gives a value of $-0{,}564$, with a std. err. of 0.216. Substituting this value into the final formula provides the short-term elasticity of substitution. The value of -0.564 indicates substitutability but at a much lower level. The value of β is also statistically significant at 1% level. The 95% conf. interval is -0.989 - -0.139.

To compute the elasticity of substitution, the following formula is used, $\beta = -(\eta - 1) \Rightarrow \eta = 1 - \beta$, Applying this, the long-term elasticity is 4.027 and the short-term elasticity is 1.564. Substitution therefore occurs in both cases, 1 percent change in relative price leads to a 4.027 percent change in the relative quantity in the long run. 1 percent change in relative price leads to a 1.56 percent change in the relative quantity in the short run.

5. Discussion

This thesis estimates the elasticity of substitution using Swedish historical data over 2 centuries. The results indicate a long-term elasticity of 4.027 and a short-term elasticity of 1.564 between woodfuel and coal. These findings shed new light on how economies historically switched between renewable and fossil energy sources. In this section, I interpret these elasticities, place them in the broader substitution literature, explore historical and technological factors that may explain the observed patterns and discuss the study's limitations along with directions for future research.

5.1 Long-term elasticity

An elasticity of substitution equal to 4.027 means that a 1 percent change in the relative price of woodfuel to coal leads to roughly a 4 percent change in the ratio of woodfuel-to-coal consumption ratio. This shows how Swedish energy consumers over two centuries have reacted strongly to shifts in relative prices and they have been able to replace wood with coal quite easily given sufficient time. Such a high long-term elasticity suggests that end-users were able to reconfigure their energy mix whenever coal became cheaper or more energy efficient.

The findings of this thesis align closely with recent findings in the modern energy-economics literature. Papageorgiou et al. (2013), using cross-country sectoral panels for the years 1995-2009, estimate long-run elasticities between clean and dirty energy inputs in the range of 2-3. Their nested CES framework indicates substantial flexibility, but my estimate of 4.027 exceeds those values, suggesting that Sweden adjusted its energy mix more rapidly. Perhaps because of its relatively uniform climate and strong industrial. Stern's (2010) meta-analysis of interfuel elasticities typically places estimates between 1 and 3, so a value of 4.027 is comparative large. The most recent study, Schwerin (2025), examines global substitution from 1800-2012 and reports a long-term elasticity of 4.33. My Sweden-specific estimate therefore aligns closely with Schwerin's aggregate findings, indicating that Sweden's historical substitution behaviour was broadly in line with global patterns. Several interrelated factors likely underpin this high long-term elasticity.

During the nineteenth and early twentieth centuries, improvements in furnace and stove design made it possible to burn coal more efficiently and safely, even on a small scale. This allowed small households to switch to cheaper fuel. In the early 1800s coal was expensive to transport and poorly suited to local needs, but this changed after the 1850s with the expansion of railways. Together with steam-powered shipping, the rail network dramatically reduced the cost of importing high-quality British coal, freeing Sweden from its limited reserves of poor-quality coal. Lower transportation costs made coal more economical than woodfuel and over the following decades its use spread beyond large industries to small towns and farm settlements.

Lower transportation costs and an economical growth in Sweden during the years 1850-1950 help explain the shift from woodfuel to coal. Rising incomes allowed households to install improved stoves and processing equipment capable of handling coal's higher energy density. Urbanisation also fostered local coal-delivery networks, letting residents buy in bulk and further reducing coal's relative price. These incentives encouraged investment in coal-fired furnaces and over time, the resulting infrastructure became so entrenched that even a fall in woodfuel prices would have needed a large additional drop in coal's price to prompt a switch back.

5.2 Short-term elasticity

The short-term elasticity of 1.564 still indicating substitutability, but with a more limited capacity to switch fuels. A value of 1.564 means that a 1 percent change in the relative price of woodfuel to coal leads to a 1.56 percent change in their relative quantities within a single year measured by the detrended data. This lower elasticity is because of several reasons and can be explained with help from literature and the long-term elasticity.

When coal suddenly became significantly cheaper in a given year, many households still relied on wood-burning stoves and fireplaces that could not handle coal. Converting a wood stove to burn coal often meant buying new parts and sometime obtaining permission because of the heavier and more intense smoke. Industries faced a similar problem and installing different mechanisms and adjusting the business could take months or even years and it could become costly.

Imperfect information about relative fuel cost also shaped household and business choices in their energy consumption. A family might buy its winter woodpile in advance, so mid-season drop in coal prices would have a small effect, especially if cash was tight. Wood was typically felled in early spring and left dry over summer, reinforcing this seasonally cycle. This type of seasonality meant that year-to-year price swings rarely produced large shifts in fuel use. The estimated short-term elasticity of 1.564 therefore shows that switching between renewable and fossil energy was not instantaneous, even when prices moved sharply

5.3 Other countries

Schwerin (2025) reports an average long-term elasticity of 4.33 for the period 1800-2012 across several economies. My Swedish estimate of 4.027 is close to this multi-country average, suggesting that no unique factor made Sweden completely atypical. This shows that Sweden shared general patterns of rapid substitution. Whit this said, Sweden's heavily reliance on forest resources may have accelerated the initial impulse to shift toward coal once transportation cost fell. It is possible that countries with even larger forest endowments, such as Canada, woodfuel may have remained relatively cheaper for a longer time, delaying early substitution.

Papageorgiou et el. (2017), using post-OECD panel data, report long-term elasticities between 2 and 3. This is lower than my estimate, but difference likely reflects the later stage of the energy transition in their sample. Once the supporting technologies are in place, fuels become less interchangeable and switching costs rise. Stern's (2010) meta-analysis, which aggregates sectoral studies of different energy sources, finds that industrial sector level often gets a value of substitution between 1.2-2.5 while macro-level estimates often fall below one. Sweden fits this pattern, woodfuel and coal were substitutable, but not perfect substitutes because it would produce an infinite elasticity. The fact that Swedish households and firms gradually replaced stoves indicates that these were easy and relative cheap changes that occurred before the later studies, helping to explain their lower elasticities. The elasticity of 4.027 are an average over a large population where the differences is substantial different. The forest was abundant in the northern Sweden but the incomes were lower. This could lead to a slower shift to coal then in the southern part of Sweden, where the transportation became more developed.

5.4 Limitations and implications

Early 19th century woodfuel data come from a Värmland household survey that Kander (2002) extrapolated backwards using a back-casting method. This approach assumes stable links between household size, income, stove efficiency and forest access across decades. Although Kander (2002) carefully documents her adjustments for improving stove efficiency and rising value of forestland, any mis-estimation of early consumption or price could bias the elasticity estimates. If actual woodfuel consumption was lower than Kander modelled, the estimated price-quantity relationship might overstate, or understate responsiveness. By running sensitivity checks, such as re-estimate the elasticity under alternative back-casting assumptions, would help provide a more balanced result.

The CES production function assumes that the elasticity between woodfuel and coal is constant across all relative price ratios and time periods which in reality is a bald assumption since substitutability likely evolved as technology developed. The usage of coal was strictly limited between 1800-1850 which made the substitution very limited during those decades. The single estimate of 4.027 therefore captures an average effect that could mask lower substitutability in early decades and higher substitutability in later decades. It would therefore be interesting to divide the sample into 50-years subperiods rather than estimating one single elasticity over 200 years. This approach would reveal if the substitutability increased alongside technological progress and trade integration or if it remained constant.

The short-term elasticity was estimated by detrending the price and quantity series with a simple log-linear trend. This is a weakness because if a structural break occurred in a specific year, it could mean that the trend cannot be fully represented by a single continuous function and the residuals could reflect long-term structural shifts rather than pure year-to-year fluctuations. There are also multiple regime changes, different unions, currency reforms and wars over the 200 years that has affected those annual fluctuations.

One key limitation of this thesis is the potential endogeneity of the relative fuel price, which means the estimated elasticity coefficient, η, may be biased due to omitted variable bias. In essence, unobserved factors could be influencing both the wood-to-coal price ratio, $(\frac{w_1}{w_2})$, and the relative consumption of woodfuel vs. coal, $\left(\frac{S_1}{S_2}\right)$, at the same time. If the price ratio is not truly exogenous and for example, if it responds to technological changes that also affect fuel use, then my

OLS regression's estimate of η will not reflect a purely causal effect. Hitrorical context offers concrete examples to this. One example is when the industrialization accelerated energy demand and introduced new technologies, likely driving both an increase in coal use and changes in fuel prices. Major wars disrupted coal supply and transportation, temporarily spiking coal prices and forcing consumers back towards woodfuel. There are also structural energy policy changes like new tariffs, subsidies or regulations favoring one fuel that could simultaneously alter the cost of fuel and the quality consumed. Technological innovations in production like more efficient steam engines or wood stoves could improve one fuel's utility while also affecting its market price. Because such variables were not included in the model, their influence can confound the relationship between relative price and consumption, biasing the elasticity estimates.

To mitigate this endogeneity problem, one would ideally control for these confounding influences if suitable data were available. For example, researchers could introduce dummy variables for war periods or other crisis year, create indicators for major policy shifts in the energy sector, or measures of energy demand such as industrial output or population growth that capture structural changes. Including lagged indicators like previous year's GDP or fuel consumption might also help account for dynamic adjustments in fuel use. Such controls would absorb some of the variation caused by external shocks or trends and thereby reducing omitted variable bias in the OLS regression. In practice, however, fully resolving endogeneity often requires more advanced econometric methods. Instrumental variable estimation is a common approach, where one finds an external instrument, a variable that affects the relative price but not fuel consumptions except through that price, to isolate exogenous price fluctuations. Similarly, structural models or dynamic panel estimators like Generalized Method of Moments (GMM) can be employed when dealing with long-term panel data.

Previous studies of historical fuel substitution have recognized this identification challenge. Kander and Stern (2014) and Schwerin (2025) explicitly address endogenous fuel prices by using such rigorous methods. In the present study, no correction for endogeneity was implemented due to data limitations, so the estimated elasticity should be interpreted with caution and as an association rather than a definitive causal parameter. Future research could improve this by applying the above methods or by identifying valid historical instrument like international coal price shocks or policy changes dictated by external events to better pin down exogenous variation in relative fuel prices. Employing these methods would yield more robust estimates of η and strengthen the causal interpretation of how relative prices influenced Sweden's wood-coal substitution.

While the CES framework provides a useful structure for estimating substitution patterns, the empirical analysis relies on historical data with known limitations in coverage and accuracy. Additionally, the regression does not fully control for all external factors that may influence both prices and fuel use, raising potential concerns about omitted variable bias. Without proper instruments or structural controls, the estimated elasticity should be interpreted as indicative rather than strictly causal.

6. Conclusion

This thesis examines how Swedish energy consumers historically switched from woodfuel to coal by estimating the elasticity of substitution over the time period 1800-2000. By combining long-run national data on woodfuel and coal consumption and modelled them with a CES production function for aggregate energy input. After linearising the CES function, I used OLS to estimate the long-term substitution elasticity. To capture short-term responsiveness, I detrended the price and quantity series to isolate year-to-year fluctuations and ran the same regression.

The main results show that woodfuel and coal were highly substitutable over the 200 years. The findings showed a long-term elasticity of 4.027, while the short-term elasticity is 1.564. This result indicates that a 1% increase in the relative price of woodfuel will result in a 4% shift in the consumption from woodfuel to coal, over decades. In the short run, a 1% increase in the relative price of woodfuel will result in a 1.56 shift in the relative consumption from woodfuel to coal. These findings align with previous global studies and imply that policy and technological change enabled a smooth historical transition in Sweden.

Overall, my analysis adds a detailed Swedish case to the literature on historical energy transitions. It provides one of the first empirical estimates of woodfuel-to-coal substitutability over two centuries in Sweden, showing how past energy policies and technical developments shaped fuel switching. These findings have implications for understanding the role of fuel flexibility in economic growth and sustainability. Future research could build on this work by examining shorter sub-periods like pre/post industrialization. It could also include other fuel types or using alternative functional forms to capture evolving technology. Such extensions would further clarify how energy substitution has changed over time and produce more robust, reliable results.

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